# Basics of Cutting Tool Geometry

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For many years there were different systems used to define a great variety of angles of faces and edges of cutting tools. Although ISO Standard *ISO 3002.1977 "Geometry of the Active Part of Cutting Tools - General Terms, Reference Systems, Tool and Working Angles"* has partially resolved this situation by definition of a system of planes and a number of angles, a number of deficiencies in definition of certain angles have been noted. The many angles defined by the standard are not necessarily independent, and many trigonometric relations describing the various angles are different from those usually developed for an acute angle of orientation of the cutting edge.

The following systems for identifying cutting tool geometry were introduced in [1]:

1. **Tool-in-Hand System**: Definitions of the basic reference planes (the main reference plane; the assumed working plane; the tool cutting edge plane; the tool back plane; the orthogonal plane; the cutting edge normal plane) for each of the cutting edges. A system of tool angles (the tool cutting edge angle; the tool minor (end) cutting edge angle; the tool approach angle; the rake angles; the clearance (flank) angles; the wedge angles; the cutting edge inclination angle), their definitions, meaning, and interrelationships among them.
2. **Tool-in-Machine System (Setting System) of Angles and Planes.**
3. **Tool-in-Use System.**

The following is to provide the definitions of some basic plane and angles in the Tool-in-Hand system.

## 1. Planes

The working part of the cutting tool basically consists of two surfaces intersecting to form the cutting edge. The surface along which the chip flows is known as the rake face or more simply as the face, and that surface which is ground back to clear the new or machined surface is known as the flank surface or simply as the flank. In the simplest yet common case the rake and flank surfaces are planes.

Figure 1 shows the definition of the main reference plane $P_r$ as perpendicular to the assumed direction of primary motion and the tool-in-hand coordinate system. In this figure, $v_f$ is the assumed direction of the cutting feed. Because angles of the cutting tool are defined in a series of reference planes, the standard defines a system of these planes in the tool-in-hand system, as shown in figure 2. The system consists of five basic planes defined relative to the reference plane $P_r$. Perpendicular to the reference plane $P_r$ and containing the assumed direction of feed motion is the assumed working plane $P_f$. The tool cutting edge plane $P_s$ is perpendicular to $P_r$, and contains the side (main) cutting edge (1-2 in Fig. 1). The tool back plane $P_p$ is perpendicular to $P_r$ and $P_f$. Perpendicular to the projection of the cutting edge into the reference plane is the orthogonal plane $P_o$. The cutting edge normal plane $P_n$ is perpendicular to the cutting edge.

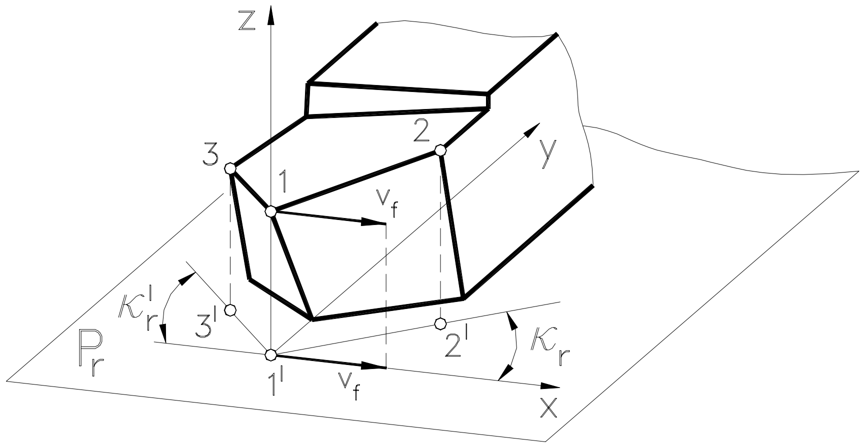



Figure 1. Definition of the main reference plane $P_r$.

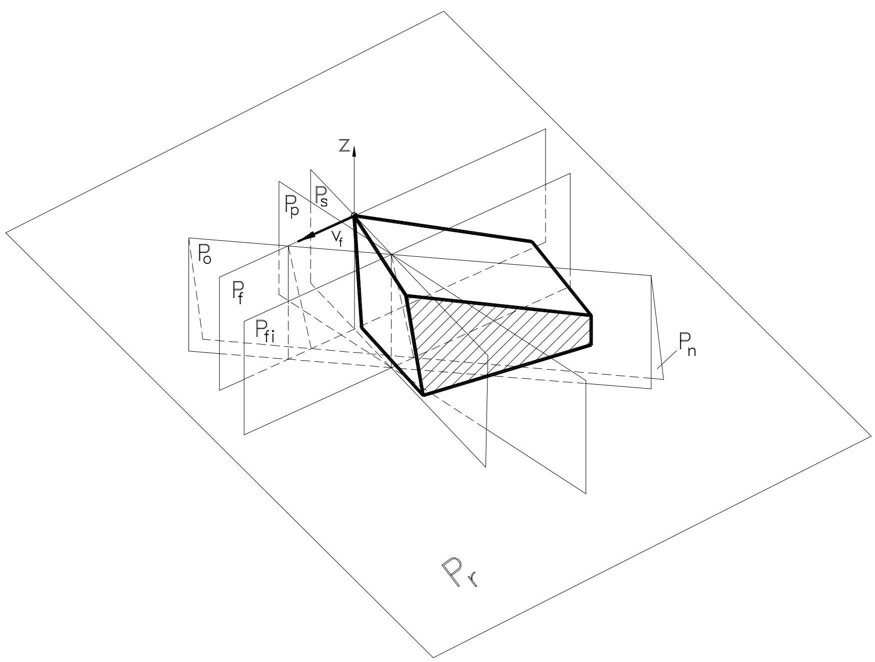



Figure 2: Standard system of reference planes in the tool-in-hand system (major cutting edge).

Similarly, an additional system of planes can be attributed to the minor cutting edge and contains the following planes: $P'_s$, $P'_o$, $P'_n$ as shown in figure 3.

## 2. Angles

The geometry of a cutting element is defined by certain basic tool angles and thus precise definitions of these angles are essential [1]. A system of tool angles is shown in figure 4. Rake, wedge and clearance (flank) angles are specified by γ, β, and

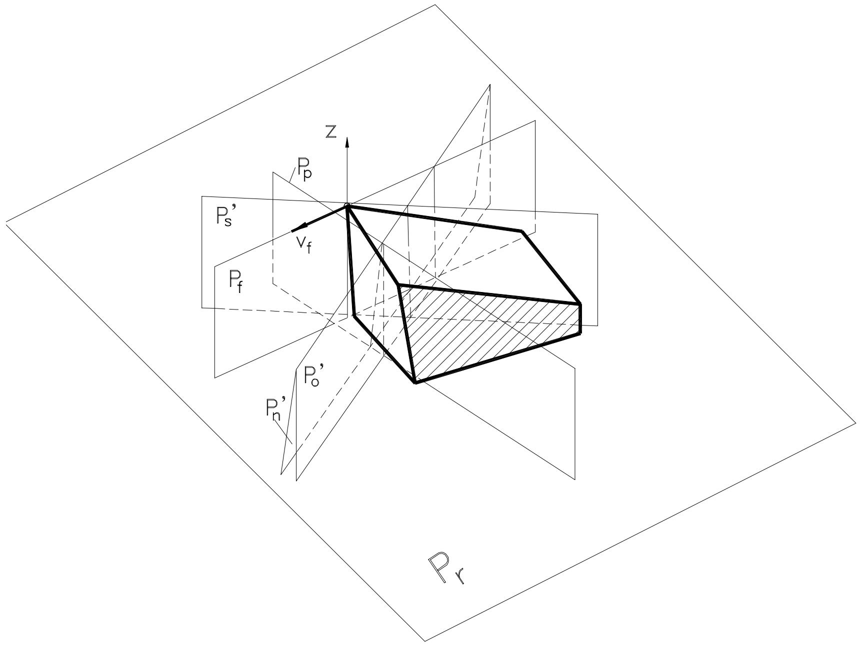



Figure 3: Standard system of reference planes in the tool-in-hand system (minor cutting edge).

$\alpha$, respectively, and these are identified by the subscript of the plane of intersection. The definitions of basic tool angles in the tool-in-hand system are as follows:

- $\kappa_r$ is the tool cutting edge angle; it is the acute angle that $P_s$ makes with $P_f$ and is measured in the reference plane $P_r$ . It can be also defined as the acute angle between the projection of the main cutting edge into the reference plane and the *x*-direction (Fig. 1). $\kappa_r$ is always positive and it is measured in a counter-clockwise direction from the position of $P_r$.
- $\kappa_{r1}$ is the tool minor (end) cutting edge angle; it is the acute angle that $P'_s$ makes with $P_f$ and is measured in the reference plane $P_r$ . It can be also defined as the acute angle between the projection of the minor (end) cutting edge into the reference plane and the *x*-direction (Fig.1). $\kappa_{r1}$ is always positive (including zero) and it is measured in a clockwise direction from the position of $P_r$.
- $\psi_r$ is the tool approach angle; it is acute angle that $P_s$ makes with $P_p$ and is measured in the reference plane $P_r$ as shown in figure 4.
- The rake angles are defined in the corresponding planes of measurement. The rake angle is the angle between the reference plane (the trace of which in the considered plane of measurement appears as the normal to the direction of primary motion) and the intersection line formed by the considered plane of measurement and the tool rake plane. The rake angle is defined as always being acute and positive when looking across the rake face from the selected point and along the line of intersection of the face and plane of measurement. The viewed line of intersection lies on the opposite side of the tool reference plane from the direction of primary motion in the measurement plane for $\gamma_f$, $\gamma_p$, $\gamma_o$, or a major

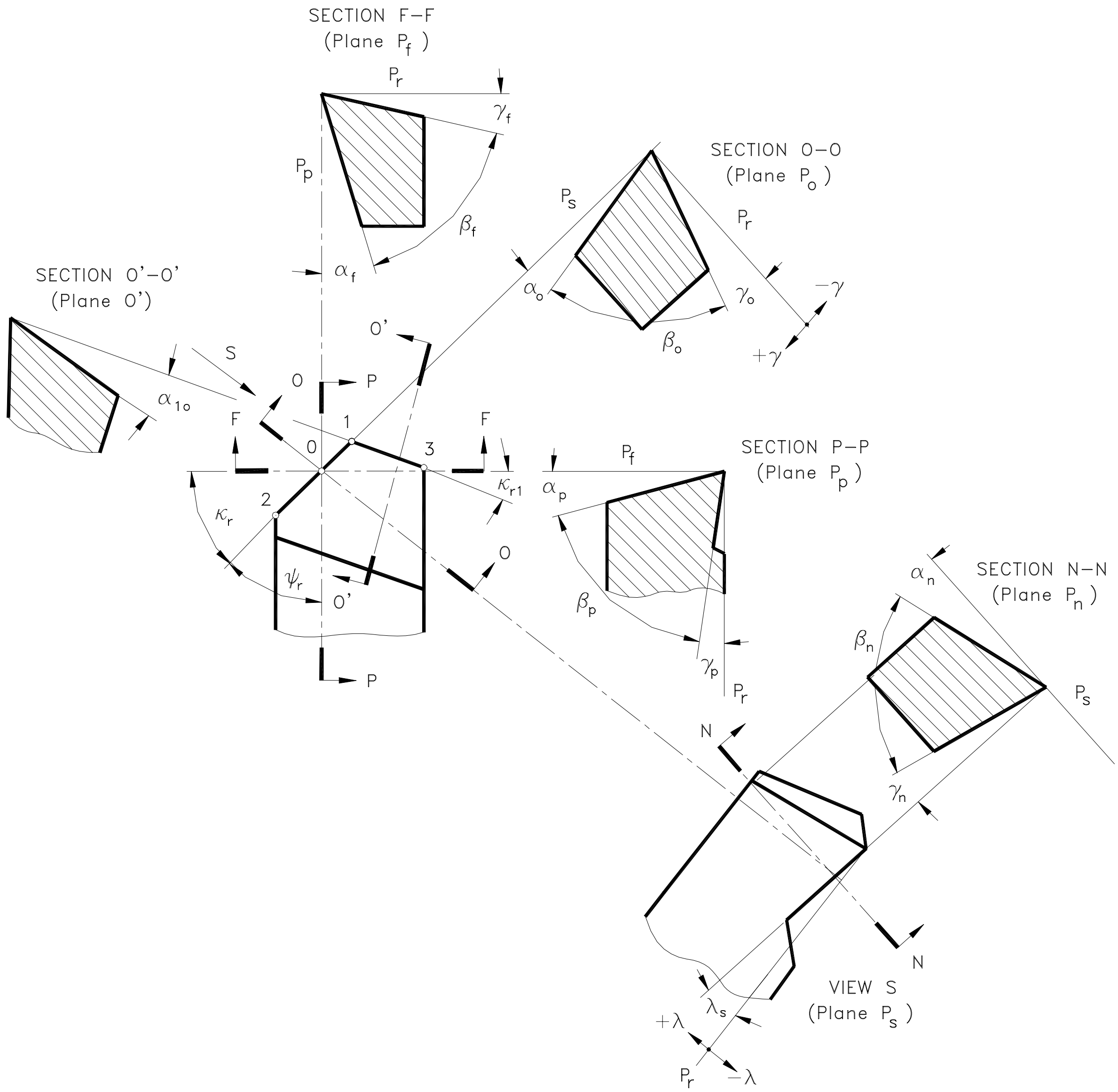



Figure 4: Tool angles in the tool-in-hand system.

component of it appears in the normal plane for $\gamma_n$. The sign of the rake angles is well defined (Fig. 4).

- The clearance (flank) angles are defined in a way similar to the rake angles, though here if the viewed line of intersection lies on the opposite side of the cutting edge plane $P_s$ from the direction of feed motion, assumed or actual as the case may be, then the clearance angle is positive. Angles $\alpha_f$, $\alpha_p$, $\alpha_o$, $\alpha_n$ are clearly defined in the corresponding planes. The clearance angle is the angle between the tool cutting edge plane $P_s$ and the intersection line formed by the tool flank plane and the considered plane of measurement as shown in figure 4.
- The wedge angles $\beta_f$, $\beta_p$, $\beta_o$, $\beta_n$ are defined in the planes of measurements. The wedge angle is the angle between the two intersection lines formed as the

corresponding plane of measurement intersects with the rake and flank planes. For all cases, the sum of the rake, wedge and clearance angles is $90^o$, i.e.

$$\gamma_p + \beta_p + \alpha_p = \gamma_n + \beta_n + \alpha_n = \gamma_o + \beta_o + \alpha_o = \gamma_f + \beta_f + \alpha_f = 90^o \qquad (1)$$

- For the minor (side) cutting edge, the flank angle $\alpha_{o1}$ is specified as the angle between the tool minor (side) cutting edge plane $P'_s$ and the intersection line formed by the tool minor flank plane and the plane of measurement $P'_o$ as shown in figure 4.
- The orientation and inclination of the cutting edge are specified in the tool cutting edge plane $P_{s.}$ In this plane, the cutting edge inclination angle $\lambda_s$ is the angle between the cutting edge and the reference plane. This angle is defines as always being acute and positive if the cutting edge, when viewed in a direction away from the selected point at the tool corner being considered, lies on the opposite side of the reference plane from the direction of primary motion. This angle can be defined at any point of the cutting edge. The sign of the inclination angle is well defined in figure 4.

Simple relationships exist among the considered angles in the tool-in-hand system. These relationships have been derived assuming that the tool side rake angle $\gamma_f$, the tool back rake angle $\gamma_p$, and the tool cutting edge angle $\kappa_r$ are the basic angles for the tool face, and the tool side clearance angle $\alpha_f$, the tool back clearance angle $\alpha_p$, and the tool cutting edge angle $\kappa_r$ are the basic angles for the tool flank [2,3]:

$$\tan\lambda_s = \sin\kappa_r \tan\gamma_p - \cos\kappa_r \tan\gamma_f \qquad (2)$$

$$\tan\gamma_n = \cos\lambda_s \tan\gamma_o \qquad (3)$$

$$\tan\gamma_o = \cos\kappa_r \tan\gamma_p + \sin\kappa_r \tan\gamma_f \qquad (4)$$

$$\cos\alpha_n = \cos\lambda_s \cot\alpha_o \qquad (5)$$

$$\cot\alpha_o = \cos\kappa_r \cot\alpha_p + \sin\kappa_r \cot\alpha_f \qquad (6)$$

It must be stated, however, that some of these relationships apply only when the cutting edge angle $\kappa_r$ is less than $90^o$. Nowadays, it is becoming common practice to use cutting tools having $\kappa_r$ greater than $90^o$. For these tools, the following relationships are valid:

$$\tan\lambda_s = -\sin\kappa_r \tan\gamma_p - \cos\kappa_r \tan\gamma_f \qquad (7)$$

$$\tan\gamma_o = \cos\lambda s \tan\gamma_o \qquad (8)$$

$$\tan\gamma_o = -\cos\kappa_r \tan\gamma_p + \sin\kappa_r \tan\gamma_f \qquad (9)$$

$$\cot\alpha_n = \cos\lambda_s \cot\alpha_o \qquad (10)$$

$$\cot\alpha_o = -\cos\kappa_r \cot\alpha_p + \sin\kappa_r \cot\alpha_f \qquad (11)$$

## 3. The Application Of Vector Analysis To The Study Of Cutting Tool Geometry

As a first step in the application of vector analysis to cutting tool geometry problems, consider the determination of the cutting edge inclination angle in the tool-in-machine system. Figures 5 shows a single-point tool having a zero inclination angle ($\lambda_s = 0$) in the tool-in-hand system (i.e., its side cutting edge1-2 is horizontal). In tool-in-machine system, the tool in installed so that its tip 1 is shifted relative to the reference plane on distance *h*. The problem is to determine the resultant cutting edge inclination angle due to this shift.

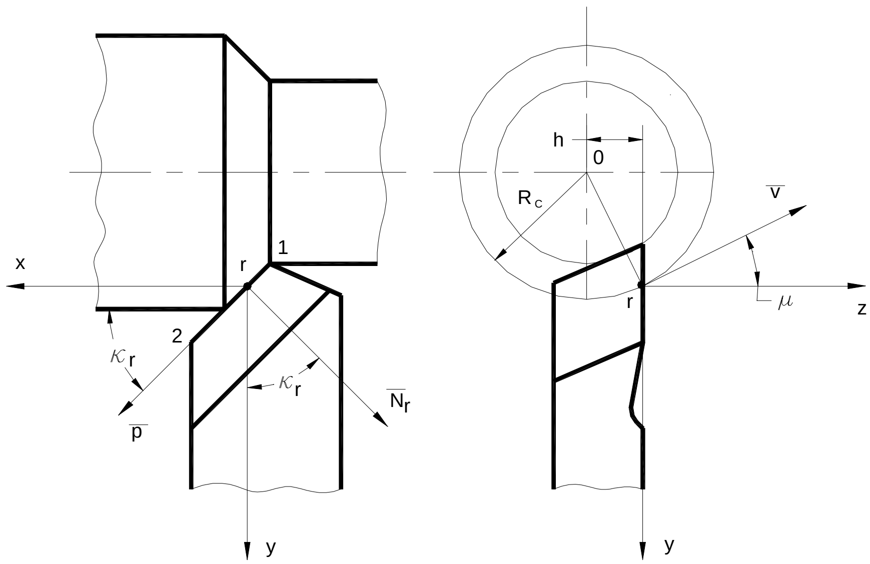



Figure 5: Geometry of a single-point tool when the cutting edge, having a zero inclination angle in the tool-in-hand system, is shifted above the reference plane through the axis of rotation. Note that the tool cutting edge angle $\kappa_r < \pi/2$.

In figure 5, a point *r* is selected on the side cutting edge 1-2 to be a current point in our consideration. The right-hand (*x, y, z*) coordinate system with the origin in point *r* is set up as follows:

1. The *x*-axis along with the feed motion.
2. The *y*-axis is chosen to be perpendicular to the *x*-axis with sense as shown in Fig. 5.
3. The *z*-axis is perpendicular to the *x*- and *y*-axes, with sense as shown in Fig. 5. Let **p** be a vector along the cutting edge 1-2 then in the selected coordinate system this vector is represented as:

$$\vec{p} = \vec{i} + \vec{j}\tan\kappa_r \qquad (12)$$

Let *h* be a distance between point *r* and the horizontal plane passing through the center of rotation *0*; $R_c$ be the radius of rotation of point *r* then the angle $\mu$ between the *z*-axis and the vector of the cutting speed at point *r* is calculated as

$$\sin\mu = \frac{h}{R_c} \qquad (13)$$

Let **v** be a vector along the direction of the cutting speed. This vector can be determined easily because it is perpendicular to *0r*, hence

$$\vec{v} = -\vec{j}\tan\mu + \vec{k} \qquad (14)$$

The angle between vectors **p** and **v** is then calculated as

$$\cos(\vec{v}\cdot\vec{p}) = \cos\left(\frac{\pi}{2} + \lambda_{sp}\right) = -\sin\lambda_{sp} = \frac{\vec{v}\cdot\vec{p}}{\left|\vec{v}\right|\cdot\left|\vec{p}\right|} \qquad (15)$$

Expressing the scalar product and vector modules through the vectors' coordinate (Eqs. 12 and 15), we obtain

$$\sin\lambda_{sp} = \frac{-\tan\mu\tan\kappa_r}{\pm\sqrt{(1+\tan^2\mu)(1+\tan^2\kappa_r)}} = \mp\sin\kappa_r\sin\mu \qquad (16)$$

Using Eq. (16) one can calculate the inclination angle for any given point of the cutting edge 1-2.

Two important conclusions may be drawn from Eq. (16). First, the inclination angle is negative (as expected according to Fig. 4) and it varies along the cutting edge due to the variation of angle $\mu$. As seen, the maximum $\lambda_{sp}$ is in point *1* and the minimum is in point 2. Second, if the tool would be installed below the discussed reference plane then angle $\mu$ would be negative. As such, $\lambda_{sp}$ is positive changing from its maximum in point 1 to its minimum at point *2*.

It was shown in [3] that the rake and clearance angles, defined in the orthogonal plane in the tool-in-hand system, change in the tool-in-machine system if a tool is shifted with respect to the reference plane. This is due to the fact that the plane of cut (which is defined as to be tangent to the surface of cut at the considered point of the cutting edge) ceases to be perpendicular to the reference plane. The angle between this plane and the cutting edge plane is denoted as $\tau_1$ and measured in the orthogonal plane. When $\lambda_s = 0$ and a cutting tool is installed as shown in Fig. 5, the rake and relief angles in the tool-in-machine system are calculated as [3]

$$\alpha_{op} = \alpha_o - \tau_1 \qquad (17)$$

$$\gamma_{op} = \gamma_o + \tau_1 \qquad (18)$$

To determine $\tau_1$, consider normal $\mathbf{N_r}$, which is perpendicular to the cutting edge in point *r* and lies in the reference plane through the cutting edge (Fig. 5). As seen

$$\overrightarrow{N_r} = -\vec{i}\tan\kappa_r + \vec{j} \qquad (19)$$

Normal $\mathbf{N_2}$ to the plane of cut is determined as the vector product of vectors **v** and **p** located in this plane

$$\vec{N}_2 = \vec{v}\times\vec{p} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -\tan\mu & 1 \\ 1 & \tan\kappa_r & 0 \end{vmatrix} = \vec{i}(-\tan\kappa_r) + \vec{j} + \vec{k}\tan\mu \quad (20)$$

Because angle $\tau_1$ is the angle between the plane of cut and the cutting edge plane, it can be calculated as the angle between normals to these planes as follows

$$\tan\tau_1 = \frac{\left|\vec{N}_r \times \vec{N}_2\right|}{(\vec{N}_r \cdot \vec{N}_2)} \quad (21)$$

The vector product of $\mathbf{N_1}$ and $\mathbf{N_2}$ is calculated as

$$\vec{N}_r \times \vec{N}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\tan\kappa_r & 1 & 0 \\ -\tan\kappa_r & 1 & \tan\mu \end{vmatrix} = \vec{i}(-\tan\mu) + \vec{j}\tan\kappa_r \tan\mu \quad (22)$$

and so its module is equal to

$$\left|\vec{N}_r \times \vec{N}_2\right| = \sqrt{\tan^2\kappa_r + \tan^2\kappa_r \tan^2\mu} = \frac{\tan\mu}{\cos\kappa_r} \quad (23)$$

The scalar product of $\mathbf{N_r}$ and $\mathbf{N_2}$ is calculated as

$$\vec{N}_r \cdot \vec{N}_2 = \tan^2\kappa_r + 1 = \frac{1}{\cos^2\kappa_r} \quad (24)$$

Substituting Eqs. (23) and (24) into (21), one can obtain

$$\tan\tau_1 = \frac{\tan\mu / \cos\kappa_r}{1 / \cos^2\kappa_r} = \tan\mu\cos\kappa_r \quad (25)$$

Analysis of Eq. (25) shows that angle $\tau_1$ varies along the cutting edge because it depends on angle $\mu$, which is a function of the radius of rotation (Eq. (13)). As a result, the rake and relief angles also vary along the cutting edge (Eqs. (17) and (19)). The maximum $\gamma_{op}$ and the minimum $\alpha_{op}$ are in point 1 while opposite is true in point 2. Moreover, $\gamma_{op} > \gamma_o$ while $\alpha_{op} < \alpha_o$. If the tool would be installed below the discussed reference plane, angle $\mu$ is negative. In effect, $\gamma_{op} < \gamma_o$ while $\alpha_{op} > \alpha_o$ and the maximum $\gamma_{op}$ and the minimum $\alpha_{op}$ are in point 2 while opposite is true in point 1.

Consider Fig. 6, which shows a single-point tool having angle $\kappa_r > \pi/2$ while other parameters and designations are kept the same. This model represents the outer

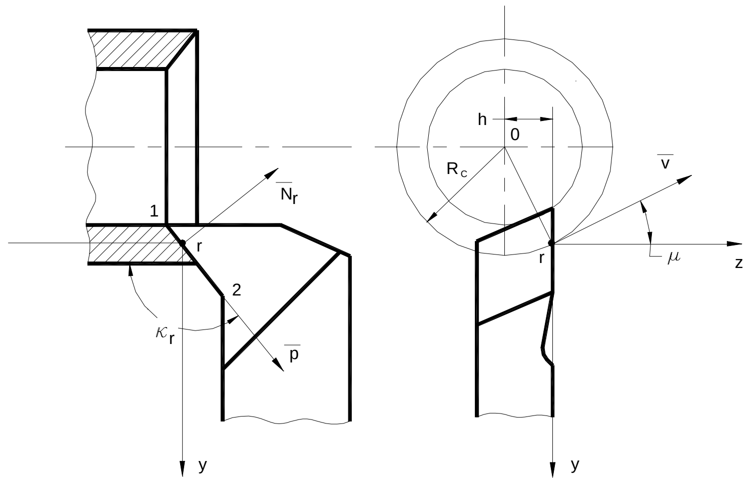



Figure 6: Geometry of a single-point tool when the cutting edge, having a zero inclination angle in the tool-in-hand system, is shifted above the reference plane through the axis of rotation. The tool cutting edge angle $\kappa_r > \pi/2$.

cutting edge of a gundrill having angle $\varphi_1 = \kappa_r - 90^o$. This angle will be used in our considerations.

Following the same methodology as discussed for Fig. 5, we can write:

Normal $\mathbf{N_r}$, which is perpendicular to the cutting edge in point *r* and lies in the reference plane through the cutting edge, is represented as

$$\vec{N}_r = -\vec{i}\tan\varphi_1 - \vec{j} \qquad (26)$$

Vectors along the cutting speed and the cutting edge, respectively

$$\vec{v} = -\vec{j}\tan\mu + \vec{k} \qquad (27)$$

$$\vec{p} = -\vec{i} + \vec{j}\tan\varphi_1 \qquad (28)$$

As before, normal $\mathbf{N_2}$ to the plane of cut is determined as the vector product of vectors **v** and **p** located in this plane

$$\vec{N}_2 = \vec{v}\times\vec{p} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -\tan\mu & 1 \\ -\tan\varphi_1 & 1 & 0 \end{vmatrix} = -\vec{i} - \vec{j}\tan\varphi_1 - \vec{k}\tan\mu\tan\varphi_1 \qquad (29)$$

The vector product of $\mathbf{N_1}$ and $\mathbf{N_2}$ and its module are calculated as

$$\vec{N}_r \times \vec{N}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\tan\varphi_1 & -1 & 0 \\ -1 & -\tan\varphi_1 & -\tan\mu\tan\varphi_1 \end{vmatrix} = \qquad (30)$$

$$= \vec{i}\tan\mu\tan^2\varphi_1 - \vec{j}\tan\varphi_1\tan\mu$$

$$\left|\vec{N}_r \times \vec{N}_2\right| = \sqrt{\tan^2\mu\tan^2\varphi_1 + \tan^2\varphi_1\tan^2\mu} = \frac{\tan\mu\tan\varphi_1}{\cos\varphi_1} \qquad (31)$$

The scalar product of **N**$_r$ and **N**$_2$ is calculated as

$$\vec{N}_r \cdot \vec{N}_2 = 1 + \tan^2\varphi_1 = \frac{1}{\cos^2\varphi_1} \qquad (32)$$

Finally

$$\tan\varphi_1 = \frac{\tan\mu\tan\varphi_1 / \cos\varphi_1}{1/\cos^2\varphi_1} = \tan\mu\sin\varphi_1 \qquad (33)$$

Analysis of the second case (Fig. 2.14) results in the following conclusions:

1. Equations (17) and (18) are no longer valid when $\kappa_r > \pi/2$, i.e., when $\kappa_r = \pi/2 + \varphi_1$. This follows from Eq. (25) where *cos* $\kappa_r$ = *cos* $(\pi/2 + \varphi_1)$ = - *sin* $\varphi_1$. It can also be seen from Eq. (29) according to which normal **N**$_2$ to the plane of cut goes 'down' (since **k** is negative) with respect to the horizontal normal **N**$_r$ (compare with the first case where this normal goes up (Eq. 22) since **k** is positive). As a result, Eqs. (17) and (18) should be re-written for the considered case as

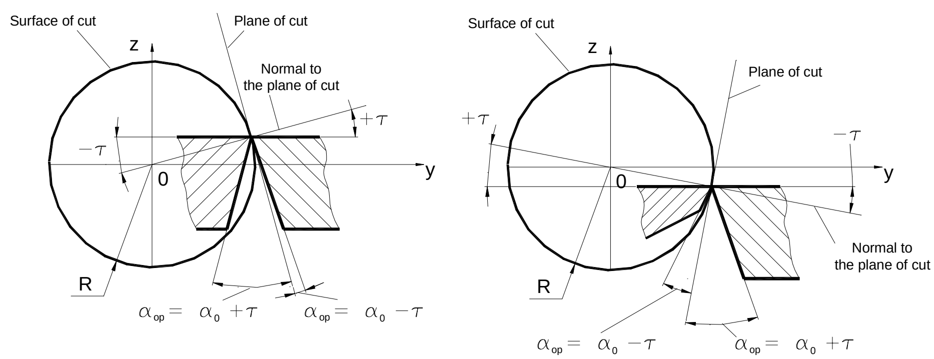
$$\alpha_{op} = \alpha_o + \tau_1 \qquad (34)$$

$$\gamma_{op} = \gamma_o - \tau_1 \qquad (35)$$

Equation (34) and (35) are of extreme importance in the considerations of the geometry of all kinds of drills because currently the opposite result (as per Eqs. (17) and (18) is used in the analysis of the rake and relief angle of twist and gun drills. The location of the cutting edge above the reference plane through the drill rotation axis leads to the increased rake and decreased flank angles if and only if $\kappa_r < \pi/2$. When $\kappa_r > \pi/2$ (and this is the common case for most drills) such a location leads to the decreased rake and increased relief angles. When the drill's cutting edge is located below the mentioned reference plane, the opposite is true. For gundrills, it leads to the increased rake and decreased relief angles on the outer cutting edge and to the decreased rake and increased relief angles on the inner cutting edge.

2. Because angle $\tau_1$ varies along the cutting, the rake and relief angles also vary along the cutting edge. The maximum $\gamma_{op}$ and the minimum $\alpha_{op}$ are in point 2 while the opposite is true in point 1.

To explain the obtained results graphically, figure 7 shows a graphical summary of the considered cases. As seen from figure 7a, when the surface of cut
is convex, any shift of the tool above the reference plane results in increase rake and decreased relief (flank) angles. However, the opposite is true when the surface of cut is concave. The latter is the case in all kinds of drilling and boring operations while the former is the case for common turning operations. As seen from figure 7b, when the surface of cut is convex, any shift of the tool below the reference plane results in decreased rake and increased relief (flank) angles. However, the opposite is true when the surface of cut is concave. When the surface of cut is plane, there is no influence of the tool vertical location on the rake and relief angles.




(a) (b)

Figure 7. Influence of the tool location on the rake and relief (flank) angles: (a) tool is shifted above the reference plane through the axis of rotation, (b) tool is shifted below the reference plane through the axis of rotation.

To complete the analysis, consider a special case when $\kappa_r = \pi/2$ as shown in figure 8. Following the same methodology as discussed for figures 5 and 6, we can write:
Normal $\mathbf{N_r}$, which is perpendicular to the cutting edge in point *r* and lies in the reference plane through the cutting edge (Fig. 8), is represented as

$$\vec{N}_r = -\vec{i} \qquad (36)$$

Vectors along the cutting speed and the cutting edge, respectively

$$\vec{v} = -\vec{j}\tan\mu + \vec{k} \qquad (37)$$

$$\vec{p} = \vec{j} \qquad (38)$$

As before, normal $\mathbf{N_2}$ to the plane of cut is determined as the vector product of vectors **v** and **p** located in this plane

$$\vec{N}_2 = \vec{v}\times\vec{p} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & -\tan\mu & 1 \\ 0 & 1 & 0 \end{vmatrix} = -\vec{i} \qquad (39)$$

The vector product of $\mathbf{N_1}$ and $\mathbf{N_2}$ and its module are calculated as

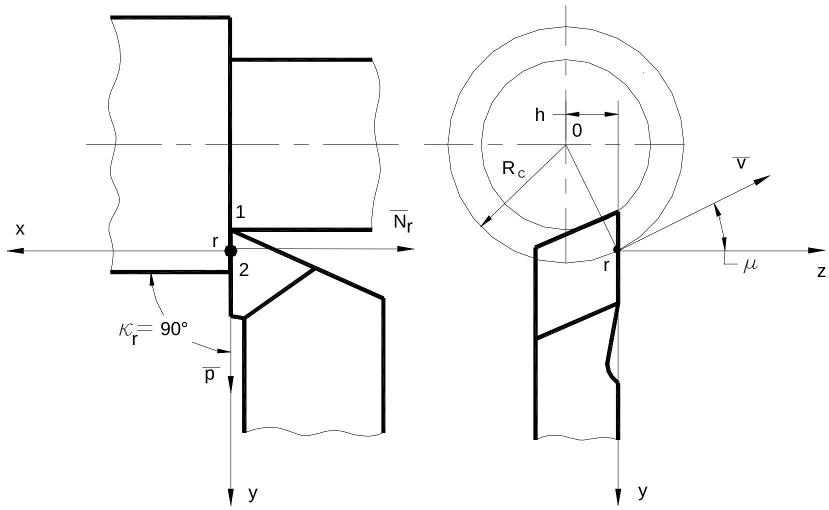



Figure 8: Geometry of a single-point tool when the cutting edge, having a zero inclination angle in the tool-in-hand system, is shifted above the reference plane through the axis of rotation. The tool cutting edge angle $\kappa_r = \pi/2$.

$$\vec{N}_r \times \vec{N}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{vmatrix} = 0 \qquad (40)$$

$$\left|\vec{N}_r \times \vec{N}_2\right| = 0 \qquad (41)$$

The scalar product of $\mathbf{N_r}$ and $\mathbf{N_2}$ is calculated as

$$\vec{N}_r \cdot \vec{N}_2 = 1 \qquad (42)$$

Finally

$$\tan\tau_1 = \frac{0}{1} = 0 \text{ thus } \tau_1 = 0 \qquad (43)$$

It follows from Eq. (43) that when $\kappa_r = \pi/2$, the vertical shift of a tool (with respect to the reference plane through the workpiece axis of rotation) does not affect the rake and flank angles of the cutting edge. The reason for this is rather simple and follows from figure 8. As seen, the surface of cut is a plane that coincides with the plane of cut and it does not change it orientation when a tool shifts along this plane.

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