METAL CUTTING THEORY - MISSED CHANCES OR A SCIENCE WITHOUT HISTORY: Part 2

Personal vision of Viktor P. Astakhov

3 "INNOVATIONS" TO THE CUTTING PROCESS

We may conclude that, unfortunately, the irrefutable above-discussed facts, most of them known since the 19th century, neither became a part of the metal cutting theory, nor influenced the thinking of subsequent further researchers. These facts were just 'conveniently' forgotten. When calculations based on the single shear model failed to match experimental results, a number of 'excuses' have been invented 'to explain' this mismatch. Imaginary high strain rates allegedly occurred in cutting along with imaginary high temperatures in the deformation zone were introduced just to save the concept. As such, a huge arsenal of material-behavior-in-cutting inventions was proposed. These calculative achievements last usually till a new experimental finding makes them inapplicable. Then new mathematical inventions are applied, instead of turning to look for the correct physical model of the metal cutting process.

This situation is similar to what it was for 1400 years, when the Ptolemaic model was reigning in science. The model was based on our very natural perception that all celestial bodies are rotating around us, as it is seen with our very eyes. This perception led to the natural concept that we on Earth are the center of the universe. The concept of "us in the center of the world" corresponded to the philosophical, religious, and ideological beliefs of those times, so that the geocentric model got full official support. To cope with this model, mathematical treatments were developed, in which planets had to rotate on hypocycles, the centers of which rotate on epicycles, the centers of which rotate on orbits around us. If the calculation results did not fit, one could always add as many rotary motions and cycles as needed for a fit. The same tendency can be traced in metal cutting.

4 PRINCIPLE OF MINIMUM ENERGY

When it was finally realized that the shear angle and thus complete solution to the single shear plane model cannot be obtained theoretically using just the above-discussed geometrical considerations, an attempt was made to apply the principle of minimum energy to calculate the shear angle. The following assumptions were made (although no one known to us reference even mentioned these assumptions):

- A single unique shear (not sliding or deformation) plane exists all the time during cutting as shown in Fig. 1.
- A unique shear velocity V_s, which is the relative velocity between the unreformed work material and the chip on the shear plane, exist along this plane, this velocity is constant, and remains the same all the time during cutting. As a consequence, a velocity diagram shown in Fig. 1 should then be adopted. Astakhov et. al, [40] showed that this velocity diagram is simply wrong because the velocity of deformation (the shear velocity) may be greater (for negative rake) than that of applied load that is physically impossible for multi-degree-of-freedom systems as the model of Fig. 15.
- The chip has chip velocity V_c and thus moves up. As such, to justify the continuity condition (Eq. (1)), the shear plane should follow the chip to prevent the separation on the shear plane. At certain instant, however, the switch between the current and successive shear plane should take place. This switch must occur instantly.
- Because the velocity of deformation on the shear plane is constant, the shearing force F_s acting along this plane and this force should remain constant over the time of existence of this shear plane. It is not clear what happens with this force during switch between the successive shear planes. If the shearing velocity V_s is constant then an infinite acceleration of this velocity should occur during this switch.
- Because the shear deformation on the shear plane is a result of material compression, the normal force F_n which is perpendicular to the shear plane should be considered. The sum of the shear and normal forces is the force R which the tool exerts on the chip. Force R may be resolved along the direction of motion of the tool relative to the workpiece into a

component F_c, the cutting force, which is responsible for the total work done in cutting, and into component F_t, the trust force, which is perpendicular to F_c.

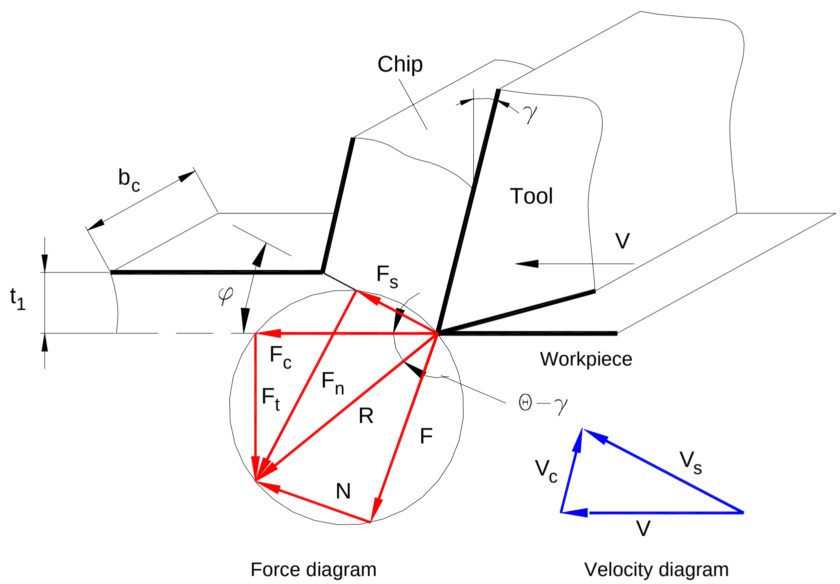

Fig. 15. Single shear plane model according to Ernst and Merchant.

According to Ernst and Merchant [33], the shearing force is calculated as

$$F_s = \frac{\tau_y A_c}{\sin\varphi} \tag{23}$$

where $A_c = t_1 \cdot b_c$ is the uncut chip cross-sectional area (Fig. 15); τ_y is the shear strength of the work material.

Accepting Eq. (23), Ernst and Merchant made one more indirect assumption, namely, the work material deforms when the stress on the shear plane reaches the shear strength of the work material. In other words, no strain hardening of the work material is allowed and thus the work material is elastic-perfectly-plastic. Although some metals may exhibit such behavior (for example, low carbon steels), the onset of plastic deformation, however, occurs at the stress which is significantly higher than the yield strength (the so-called upper yield point) so that the shearing force F_s cannot be constant even for these specific materials. For the steels, used in the experiments Ernst and Merchant [33], strain-hardening takes place and thus the so-called flow shear stress is much higher than the yield strength of the work material.

Besides, it was also assumed that the flow shear stress on the shear plane does not depend on the normal stress on this plane.

It follows from Fig. 15, that

$$F_c = \frac{F_s \cos(\theta-\gamma)}{\cos(\varphi+\theta-\gamma)} \tag{24}$$

and using Eq. (23) one can obtain

$$F_c = \frac{\tau_y A_c \cos(\theta-\gamma)}{\sin\varphi\cos(\varphi+\theta-\gamma)} \tag{25}$$

The work spent in cutting is calculated as

$$U_c = F_c V \tag{26}$$

where V is the cutting velocity (Fig. 15).

According to the principle of minimum work, the work done in cutting should be at minimum, and thus its derivative should be zero, i.e.

$$\frac{\partial}{\partial\varphi}(U)=\frac{\partial}{\partial\varphi}(F_c V)=V\frac{\partial}{\partial\varphi}(F_c)=0 \tag{27}$$

Because the cutting velocity V is not equal to zero, it follows from Eq. (5) that

$$\frac{\partial}{\partial\varphi}(F_c)=0 \tag{28}$$

Analyzing Eq. (25), Ernst and Merchant concluded that $\tau_y A_c \cos(\theta - \gamma)$ is constant. Accepting this fact one should realize that another assumption is made, namely, that the angle of friction θ and thus the friction coefficient is constant at the tool/chip interface that was not proven to be the case by many studies (this will be discussed later in this chapter).

Substituting Eq. (25) and accounting for the assumption that $\tau_y A_c \cos(\theta - \gamma)$ is non-zero constant one can obtain

$$\frac{\partial}{\partial\varphi}(F_c)=\frac{\partial}{\partial\varphi}\left(\frac{\tau_y A_c\cos(\theta-\gamma)}{\sin\varphi\cos(\varphi+\theta-\gamma)}\right)=\tau_y A_c\cos(\theta-\gamma)\frac{\partial}{\partial\varphi}\left(\frac{1}{\sin\varphi\cos(\varphi+\theta-\gamma)}\right)=0 \tag{29}$$

from which the above-discussed (Eq. (18) Ernst and Merchant solution is obtained

$$2\varphi_{sp}+\theta-\gamma=\frac{\pi}{2} \tag{30}$$

Having conducted a number of experiments, Merchant realized that, unfortunately, this solution is in very poor agreement with experimental results. Instead of re-thinking the severe above-discussed assumptions made in developing the single shear plane model, Merchant, however, tried to 'adjust' the behavior of the work material suggesting that the shear strength of the work material depends linearly on the normal stress on the shear plane (Eq. (22). He thought that that this might account for the fact that his experimental results were not in agreement with Eq. (30). Combining Eq. (22) and Eq. (25) it follows that

$$F_c=\frac{\tau_y A_c\cos(\theta-\gamma)}{\sin\varphi\cos(\varphi+\theta-\gamma)[1-k_1\tan(\varphi+\theta-\gamma)]} \tag{31}$$

Applying the principle of minimum energy and following the same procedure as before, that is, minimizing F_c with respect to φ in order to obtain an expression for φ, it can be readily shown that

$$\begin{cases}2\varphi+\theta-\gamma=c_1\\ c_1=\cot^{-1}k_1\end{cases} \tag{32}$$

and this solution is known as the modified Merchant solution which was obtained earlier (Eq.(19)) on the basis of pure geometrical considerations. As discussed above, there is still a marked disagreement between this solution and the experimental results.

The subsequent researchers had to deal with this problem. Most of them refused to admit the discussed failure ignoring the known discrepancy. For example, in the recent publication by Shamoto and Altintas [41] the Ernst and Merchant approach was applied to oblique cutting. It was proudly mentioned by the authors that when three-dimensional oblique cutting model is applied to two dimensional orthogonal cutting, it yields the same shear angle expression as Eq. (30).

Some researchers tried to understand the nature of the discussed discrepancy between the theoretical and the experimental results. Because the single shear plane model (and thus all above mentioned assumptions) was a 'holy cow' and thus should not be questioned, the principle of minimum energy came under fire. The signal was shown by Bishop and McDougall [36,42] who wrote that "the minimum force principle as used in the actual theories is at best **a hypothesis** and as actually employed is not valid." Full support to this point was given by Hill [42]. Their criticism, however, was of qualitative rather than quantitative nature: if the experimental results do not match the theoretical obtained using the principle of minimum energy, this principle is not valid at all. <u>Bishop and McDougall, and Hill did not pay much attention that Ernst and Merchant applying this principle to the one model got one result and then Merchant applied the same principle to the modified model and got a different result</u>.

The most 'convincing' result on inadmissibility of applying the principle of minimum energy was presented by Rubenstein [43]. Rubenstein proposed a method of testing the admissibility of this principle in metal cutting. The essence of this method is to test the experimental results against the theoretical obtained by minimizing the following expression for the cutting force F_c

$$F_c = F_c^1 + \tau t_1 b_c (\cot\varphi + 1) \tag{33}$$

where F_c^1 is the force component arising from the inevitable flow of a thin layer of the work material below the tool (according to Rubenstein, this component is determined by the cutting edge radius, the tool clearance angle, and the degree of adhesion (probably a new parameter introduced by Rubenstein, *auth.*) between the tool and the workpiece); τ is the shear stress acting on the lower boundary of the primary deformation zone; φ is "the Merchant shear angle".

Analyzing Eq. (33) one may wonder what it has to do with the single shear plane model where the tool is considered to be perfectly sharp so that no ploughing can occur and thus the plowing component F_c^1 (Rubenstein did not use this term, although it was considered in details almost twenty years earlier by Albrecht [44]) should not be considered; the rake and friction angles are not even included; the deformation zone instead of a single shear plane is considered. No wonder Rubensteing obtained a very strange result differentiating Eq. (33) and thus declared that 'of the minimum work hypothesis' is inapplicable in metal cutting.

To finish the discussion on the principle of minimum energy, it is worthwhile to point out the following. This principle (not hypothesis) of minimum energy (not work, force, etc.) is one of the most fundamental principles of physics (like gravity, inertia, etc). It is applicable to any physically realizable technical system so that it must be applicable in metal cutting where no paranormal activities were yet observed. If there is a discrepancy between the theoretical and experimental results, the researches should blame the incorrect model used in studies. Unfortunately, this is not the case in metal cutting where the single shear plane model is still untouchable.

5 THEORY DUE TO LEE AND SHAFER

Ernst and Merchant in their studies never considered the state of stress (and strain) in the work material ahead of the tool, in the chip, and at the tool/chip interface. In other words, it was not their concern to understand how the force exerted by the tool rake face is transmitted to the shear plane.

The first attempt to solve this problem was made by Lee and Shafer [35]. For their modeling, Lee and Shafer considered the following:

- <u>Method of machining</u>: orthogonal cutting as the simplest case.
- <u>Work material</u>: a rigid-perfectly plastic solid (no work hardening was allowed). It was considered as a good approximation for steels and cast irons for three reasons [45-47]. First, work hardening of steels decreases rapidly with increasing strain and this in the large strains, which occur in metal cutting, the material yields at a fairly constant stress for most of the strains. Second, since the effect of <u>high strain rate</u> is to raise the yield strength of a material with respect of its ultimate stress, then at high strain rates of the order of $10^5\ s^{-1}$ which exists in the cutting operation, the stress-strain curve of the work material would tend to that of a non-work hardening material. Third, since the elastic yield-point strain of most materials is of the order 10^{-3}, it can be safety neglected in analyzing large plastic deformations.
- <u>State of strain</u>: plane strain conditions because it was observed that no sideways spread occurs (apart from an edge effect).

We have to point out here that the assumptions made about the work material are very questionable. Although the first reasoning about <u>work hardening</u> may be partially applicable to very mild steels, the second assumption about high strain rates in metal cutting has no ground because no one research was carried out to measure this rate experimentally [31] prior Lee and Shafer study. Therefore, their main argument about work material behavior is the weakest one.

Lee and Shafer were the first who tried to apply engineering plasticity (vigorously developing in the same years by Hill [48]) to solve the metal cutting problem. In engineering plasticity, the solution of <u>this type of problems involves the following distinctive stages</u>:

1. Determination of stress distribution, which must satisfy the boundary and equilibrium conditions as well as the accepted yield criterion.
2. Obtaining the displacement (velocity) solution, which must satisfy the boundary and incompressibility conditions.

3. Verifying the consistency of the stress and displacement solutions bearing in mind that for a perfectly plastic material stress and plastic strain are not related by a material constant.

The method of obtaining these solutions proposed by Hill [48] and used by Lee and Shafer includes a determination of the system of orthogonal curvilinear trajectories of the maximum shear stress, called slip lines, which defines the region of plastic deformation where the material is stressed to the yield.

Figure 16a shows an infinitesimal element at any point of the region of plastic deformation under plane strain conditions. As seen, each face of this element is subjected to a normal stress σ_x or σ_y and a shear stress τ_{xy} or τ_{yx}. This is true for all orientations of the considered element except one shown in Fig. 16b, where there are no shear stresses. The normal stresses are then called principal stresses, σ_1 and σ_2 and their directions are principal directions. At 45^o to this orientation (Fig. 16c) the shear stresses *k* are at their maximum and the normal stresses, *p* are equal. Although, the maximum shear stress *k* is constant for a non-workhardening material depending only on its yield strength, the direction of *k* varies from point to point. If the directions of maximum *k* were traced out for the entire region of plastic deformation, they would form a net (or field) of orthogonal curved trajectories. These are the slip lines.

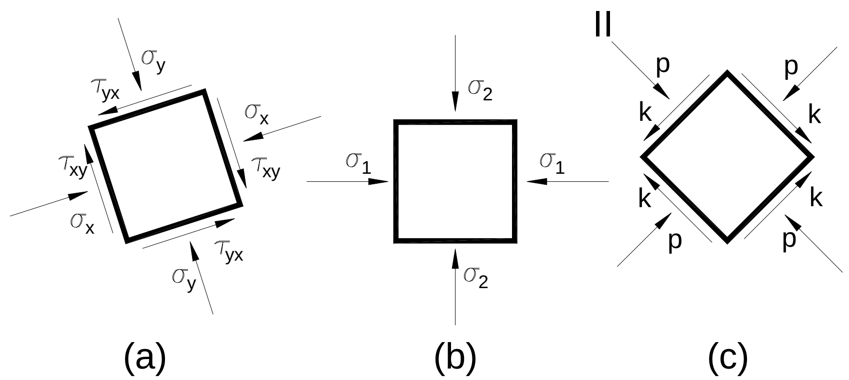

Fig. 16. **Equivalent stress system on a small element in the plane of deformation under conditions of plane strain: (a) stresses on an element with general orientation; (b) principal stresses; (c) maximum shear stresses.**

The normal stress *p* is also varies from point to point in the region of plastic deformation. When the slip-line field has been determined, the value of *p* can be determined using the Hencky equations

$$p + 2k\xi_I = \text{constant along the I slip line} \quad (34)$$

$$p + 2k\xi_{II} = \text{constant along the II slip line} \quad (35)$$

where ξ_I and ξ_{II} are the angles between the tangent to the slip lines I and II at a point of interest and a fixed direction (for example, the *x*-axis [9]).

Similar considerations apply to the state of strain at a point of interest. Incompressibility of the work material and plane strain condition yield

$$\varepsilon_I + \varepsilon_{II} = 0 \quad (36)$$

Because there is no change in length along the slip lines, Lee and Shafer concluded that the work material is subjected to pure strain in the plastic zone. We have to point out here that this is true if and only if there is no velocity discontinuity is allowed in the deformation zone. Unfortunately, Lee and Shafer as well as the subsequent studies did not pay attention to this fact assuming that pure shear strain deformation takes place also in the single shear plane model and in the model with parallel boundaries where velocity discontinuity cannot be ignored [9].

The slip line field gives the instantaneous state of yielding and so it is suitable for the solution of such problems as, for example, indentation. In metal cutting, however, as deformation proceeds, the slip-line field changes and so a solution of such a problem has to be carried out in a step-by-step manner of the changes if the shape of the workpiece and the chip are to be allowed. Because there were no computers readily available for such cumbersome procedure at the time of the Lee and Shafer study, the problem of orthogonal cutting was treated as ‘steady state’ problem where the shape of deforming region remains constant because it is predetermined by the shape of the tool and hence the slip-line field remains unchanged. Since then, orthogonal metal cutting is treated as one of the deforming processes (extrusion, rolling, drawing) ignoring obvious facts:

- In any deforming process, the workpiece changes its shape due to plastic deformation preserving its volume. In metal cutting, the chip physically separates from the rest of the workpiece so that the workpiece changes its volume. There are two separate items after machining, namely, the machined workpiece and the chip.
- The chip has always segmented structure [9]. The steady-state solution does not allow the switch between the fragments.

Attempting to apply the discussed basics of slip-line method, Lee and Shafer developed their model of chip formation shown in Fig. 17 where the workpiece is stationary and the tool moves from right to left. Because a steady-state problem was kept in mind, the model was considered when the tool advances far enough that the stress in the deformation zone reaches the yield point. The shear plane *AC* separates the undeformed work material from the chip. Its orientation was chosen to give the minimum machining force. As a result, when an infinitesimal element of work material crosses the shear plane *AC*, it rapidly sheared into the chip. In this process it acquires that the velocity, which makes it move up the tool and the stress falls to zero. This rapid onset of deformation and velocity occurs at the edge *AC* of the slip-line field *ABC* which was deduced by Lee and Shafer to be as shown in Fig. 17. For such a model, the constructed slip-line field is fictitious because no plastic deformation is allowed above the shear plane *AC* . In our opinion, a slip-line field makes sense if and only if it is used to describe strain development in a deformation zone. The model considered by Lee and Shafer (Fig. 17) where no plastic deformation was allowed in the slip-line field has no physical difference from that by Ernst and Merchant [33].

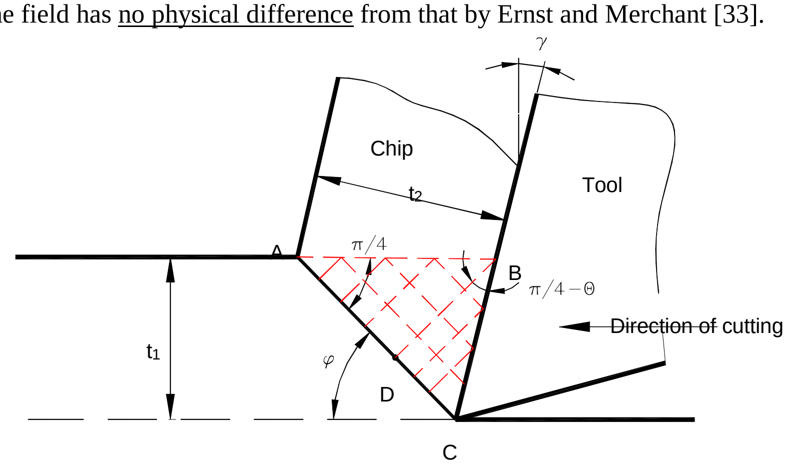

Fig. 17. Lee and Shafer’s slip-line field solution for no built-up edge.

Because the state of stress in *ABC* is uniform, a Mohr’s circle diagram was used for its representation as shown in Fig. 18. Since the maximum stress that can be withstood is $k = \tau_y$, the radius of the circle is *k*. Further, since there is no force across *AB* then the Mohr circle passes through the origin. There are two possible circles satisfying these conditions, one in which all normal stress components are tensile and the other in which all are compressive. The latter is obviously the appropriate one. The normal stress on the slip lines is determined by the abscissa of the center of the circle to be equal $p = k$. The shear stress thus is $\pm k$.

Considering the equilibrium of triangle *DCB*, Lee and Shafer concluded that the slip lines (as the lines of maximum shear) meet the tool face at angles $DCB = \eta$ where $\eta = 45^o - \theta$, and so the lower boundary *AC* of the slip-line field meets the tool face at an angle $(90^o - \eta)$. Further, after passing the slip-line field the chip is subjected to no forces and so no stresses is transmitted across the upper boundary *AB*. Therefore, the slip lines must meet *AB* at 45^o. Using this consideration, a location of point *e* on the Mohr circle was determined by the angle η (Fig. 18). Having determined this location, Lee and Shafer used the similarity of the Mohr circle to determine the friction angle θ at the tool/chip interface as shown in Fig. 18.

Therefore, the friction coefficient μ = tan θ was defined as the ratio of the shear and normal stresses at the tool/chip interface. In our opinion, this is a very important deduction, which unfortunately did not attract any attention of the subsequent researches who unanalyzed the Lee and Shafer model.

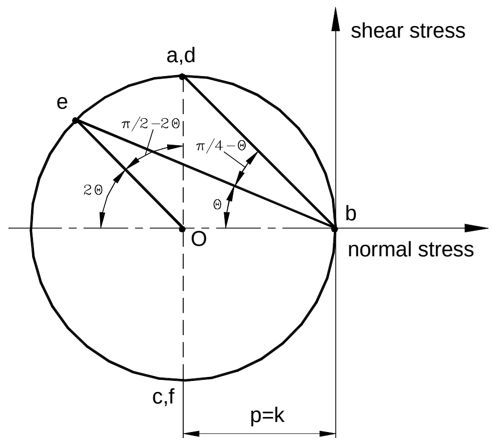

Fig. 18. Mohr's circle diagram for Lee and Shafer's solution with no built-up edge.

The so constructed slip-line fields consisting of an orthogonal set of straight lines (Fig. 17) satisfies the above-discussed conditions. If now the Hencky equations (Eqs. (34) and (35) are applied, it may be seen that the stress distribution in *ABC* is uniform. These results end the first distinctive stage of the solution.

As mentioned above, the second stage must be a corresponding displacement (velocity) solution consistent with this slip-line field. By considering the relative motion of the tool and work material, it can be shown that the velocity components of an infinitesimal element of work material crossing the slip lines are constants. As a result, it was concluded that the whole slip-line field *ABC* moves as a rigid body. Across *AC* there is a discontinuity in the tangential component of velocity that results in infinite shear strain rate and, therefore, as in Ernst and Merchant solution, shear takes place only on one plane, namely the shear plane *AC*. A logical question rises then about the contracted slip-line field where no deformation is allowed.

From the pure geometrical considerations, the Lee and Shafer solution gives the following expression for the shear angle

$$\varphi + \theta - \gamma = 45 \tag{37}$$

The horizontal (power) component of the cutting force was obtained by considering the equilibrium of region *ABC* (Fig. 17). Because the surface *AB* is the stress free, the force on the tool is equal to the force on surface *AC*. Further, as accepted in the model, the shearing and normal stresses on *AC* are both equal to *k*. It can therefore be shown that

$$F_c = \tau_y t_1 b_c (1 + \cot\varphi) = \tau_y t_1 b_c \left(1 + \cot\left[\frac{\pi}{4} - \theta + \gamma\right]\right) \tag{38}$$

We have to point out here that b_c being the width of cut is somehow omitted in the equation for the horizontal component presented by Lee and Shafer in [35]. Pugh [36] analyzing the Lee and Shafer model just repeated this equation without any comments.

The chip thickness ratio (this term was used by Lee and Shafer [35]) can then be calculated as

$$r_c = \frac{t_2}{t_1} = \frac{\sin\left(\frac{\pi}{4} - \theta + \gamma\right)}{\cos\left(\frac{\pi}{4} - \theta\right)} \tag{39}$$

It is interesting to note that Eq. (39) represents the chip compression ratio as reciprocal to the chip thickness ratio used by Merchant [34]. Unfortunately, this difference was not pointed out in the subsequent studies where the Lee and Shafer model was considered in details [36].

It may be seen from consideration of Eqs. (38) and (39) that if γ is negative, say -10^o, and with, say $\theta = 35^o$, $\cot(\pi/4 - \theta + \gamma)$ becomes infinite so do F_c and t_2. To resolve this problem, Lee and Shafer tried to work out a slip-line field based on the idea of the built-up tool nose.

The Lee and Shafer model with the built-up tool nose is shown in Fig. 19 and consists of a small centered fan-like slip-line field to give a built-up nose in conjunction with the uniform stress field of the previously considered solution. Constricting this model, however, Lee and Shafer made so many unjustifiable assumptions that one might have a hard time understanding their considerations. Among these assumptions are:

- The model is developed for high friction and small or negative rake angle. The angle ϕ of the fan-like slip line field is derived as

$$\phi = \frac{\pi}{4} - \psi + \theta - \gamma \tag{40}$$

where ψ is angle *CFE* on Fig. 19 (which cannot be determined because the side *FC* is curved). Now, if we substitute $\gamma = -\pi/8$, assume (using the Lee and Shafer data provided in Fig. 5 or 6 in [35]) $\mu = 1.73$, then $\theta = \pi/3$. Then from Eq. (40) it follows that

$$\phi + \psi = 0.705\pi = 126.9^o \tag{41}$$

In the model development, angle ϕ should be small and thus the proposed configuration of the built-up edge cannot physically exist.

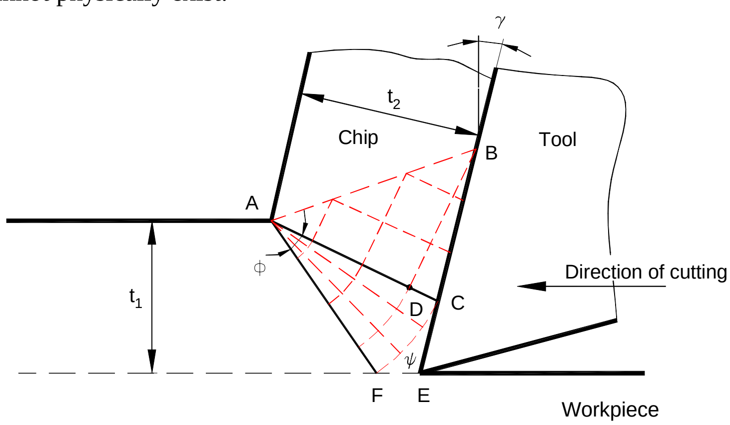

Fig. 19. Lee and Shafer's slip-line field configuration with the built-up edge.

- The rigid region *CFE* takes a form of a built-up nose on the face of the tool and it is considered to be in the limiting state of stress near apex *F*. However, if the work material is perfectly plastic, how can edge *F* cut the same material, which, according to Lee and Shafer is under the same stressed state. It should be clear that tool material must be harder that the work material. Moreover, in the consideration of the force on this built-up nose, it is stated that it must be 'stresses only elastically' that physically is not realizable.
- The normal stress on slip line *AF* is assumed to be equal to $k/(1+2\theta)$ without any explanations. Lee and Shafer just mentioned that they took this equation from Hill's work [49]. As it is known now [50-52], this is valid only for a certain geometry of die and billet in extrusion or in upsetting and thus cannot be considered as the general case.
- To construct the Mohr circle corresponding to the modified model, the angle of friction along *FE* should be know (up-to-date there is no reliable method to measure this angle). Moreover, this angle is forgotten in the equation for the friction force on *FE* (Eq. (11) in [35]) where the friction angle peculiar to the tool/chip interface is used.
- The angle of friction at the chip/built-up interface (surface *FC*) is assumed to be the same as at the tool chip interface.

As a result of these assumptions, the modified model was found to be even less practical than the first one [36].

6 UNIQUENESS OF MACHINING

An important step in the development of engineering plasticity was a book by Hill [48] where he formulated the known Hill's limit theorems. According to Hill, since the stress distribution in both the single shear plane model and the Lee and Shafer model [35] are only given for a part of the body, then the corresponding solutions (the so-called type 1 and type 2) give an upper limit to the yield point loads, i.e., they are only upper-bound solutions. It also follows from these limit theorems that, of two upper-bound solutions, the one, which gives the greater yield point load, should be rejected as a possible solution. Bishop and McDougall [36,42] managed to prove that in cutting, however, when $\theta \leq \gamma$, both the type 1 and type 2 solutions are valid upper-bound solutions. Moreover, they are solutions of different problems. On the basis of their analysis, Bishop and McDougall concluded that in the cutting process the deformation mode is not determined directly from the problem and that it may well depend upon the conditions existing from the commencement of cutting. In simple language it means that no unique solution to the cutting problem exists.

Following on this work, Hill [42] suggested that the search for a unique steady-state solution from the machining problem was probably fruitless and that there may be many complete steady-state solutions of a given type such as the single shear plane type.

Since there is so little constraint on the flow of the material in machining process, it seems highly probable that the ultimate steady flow be influenced by the initial conditions. In this event, the problem reduces to that of determining a range of steady-state solutions each linked by an intervening non-steady flow with given initial conditions. Hill has made a start on this new approach by establishing for the complete shear plane type of solution the extent of the <u>permissible range of shear plane angles</u>.

In order to determine the excluded ranges in which the shear plane angles are not admissible, Hill considered <u>the single shear plane model</u> shown in Fig. 20 where a wedge-shaped tool is moved in the

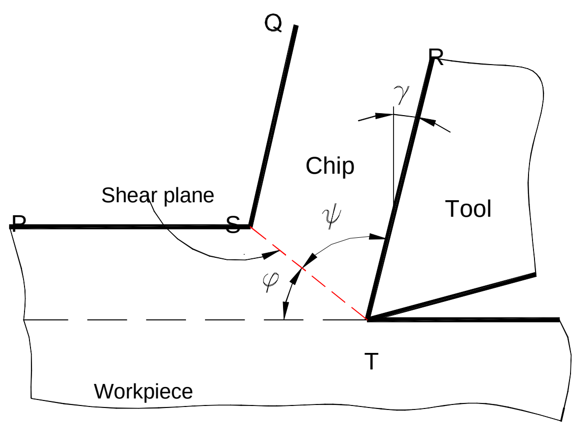

Fig. 20. Hill's model.

direction normal to its cutting edge and parallel to the plane surface of the workpiece. The depth of cut is assumed small compared to its width. Using this model, Hill investigated the possibility of steady states in which the zone of deformation is concentrated in a single plane (*ST*) springing from the tip of the tool (*T*). The inclination of this shear plane to the surface of the workpiece is denoted as φ and its inclination to the tool face by ψ. Adopting this model, Hill made the following indirect assumptions that might have greatly affected the results of his study:

- A unique shear plane having "steady-state" inclination angle φ exists all the time during the cutting process. No switch between successive shear planes is allowed.
- An idealized rigid-perfectly plastic work material is the case.
- All the deformation of the work material takes place along the shear plane. When one considers the single shear plane geometrically (note that there is no physics or mechanics of materials involved in earlier considerations), as everybody did before Hill, such an approximation does not fight back. In mathematics, however, it leads to the so-called stress and deformation singularities.

Moreover, Hill assumed additionally that the side *SO* of the chip free surface is parallel to the rake face *TR* and simple Coulomb friction having invariable friction angle θ takes place along the tool/chip interface. In our opinion, the last two assumptions just put the final nail to the coffin of the whole study, which is full of unreasonable assumptions. The reason for this it is quite simple: Hill was not a specialist in metal cutting or in the physics of materials. Engineering plasticity greatly developed by Hill is a branch of mathematics, which considers a real material as "perfect solid" having no structure, anisotropy, etc.

What is great about Hill's study is that, because Hill tried to use properly the method of engineering plasticity, he recognized for the first time that the velocity is tangentially discontinuous across the shear-plane. Unfortunately Oxley refused to admit this self-obvious fact (though may be for those who understand engineering plasticity and the mathematics involved) twenty-five years later. Due to this discontinuity, the shear plane must be in the direction of maximum shear stress, τ_{max}. Moreover, by Hencky's theorem, the pressure acting across the shear plane is uniformly distributed with magnitude p_{sh}. From Fig. 20, this pressure is

$$p_{sh} = \tau_{\max}\cot(\psi - \theta)\text{, with } \psi > \theta \tag{42}$$

because the resultant traction across the shear plane must balance the force exerted by the tool (no other forces being applied to the chip). Following the approach, proposed by Bishop and MaDougall, Hill considered simple static equilibrium within each of three angles *PST*, *QST*, and *RTS* (Fig. 21) assuming that

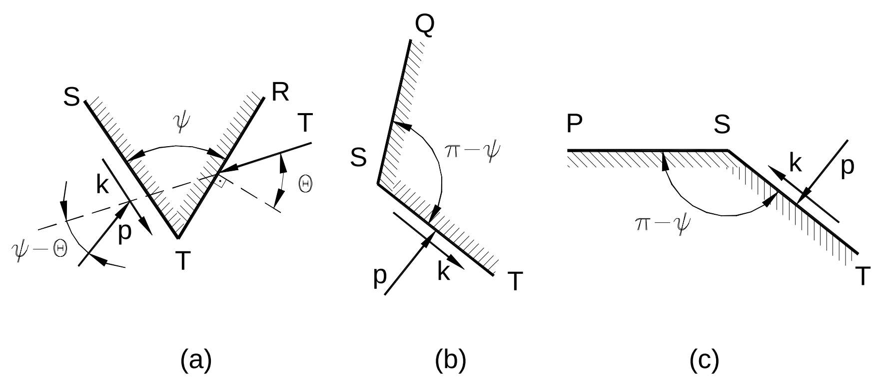

Fig. 21. Static equilibrium models.

the shear stress in the work material cannot be greater than *k* and the normal stress across the shear plane *p* cannot exceed *k*. Hill's analysis yielded in the following

$$\theta < \psi \le \frac{1}{4}\pi + \theta, \qquad \text{if} \qquad 0 \le \theta \le \frac{1}{4}\pi \tag{43}$$

$$\theta < \psi \le \frac{1}{2}\pi, \qquad \text{if} \qquad \theta > \frac{1}{4}\pi \tag{44}$$

and

$$\frac{p}{k} = \cot(\psi - \theta) < 2\sin\left(\psi - \frac{\pi}{4}\right) - 1 \ \text{ if } \ \psi > \frac{3}{4}\pi \tag{45}$$

$$\frac{p}{k} = \cot(\psi - \theta) > 1 - 2\sin\left(\varphi - \frac{\pi}{4}\right) \ \text{ if otherwise} \tag{46}$$

The values of φ and ψ not excluded by Eqs. (43) - (46) are shown as the shaded area in Fig. 22a. This is schematic diagram for some given value of θ which was taken by Hill less than 1/4π. Taking sections of Fig. 22a by lines φ + ψ = 1/2π + γ for various γ (positive and negative), Hill was able to plot φ against θ - γ as shown in Fig. 22b which was customarily in metal cutting papers. Analyzing a batch of similar diagrams plotted for different friction angles, Hill concluded that no steady configuration of the shear plane model is possible.

The next (and the last so far) significant step in the analysis of the non-uniqueness of the machining process was taken by Dewhurst [53] who tried to incorporate all new finding in engineering

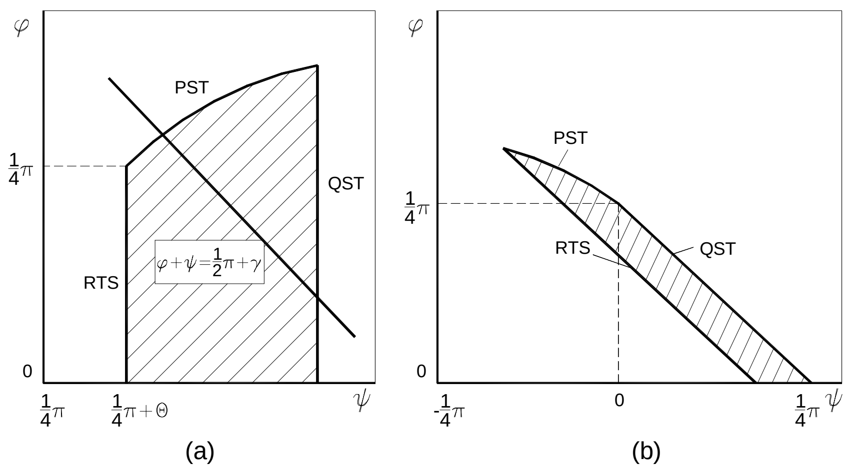

Fig. 22. Values of the shear and rake angles not excluded by the equilibrium equations.

plasticity and metal cutting mechanics to construct the model of chip formation. Analyzing state of the art in the mechanics of machining, Dewhurst noted that the current investigators, notably Oxley and coworkers, have taken the view that an understanding of the process can only be attained by considering the distribution of temperature in the deformation zone as well as the influence of strain-rate and strain hardening of the deforming material. With this premise however the problem becomes analytically intractable so that theoretically unjustifiable assumptions about the mode of deformation are needed in order to obtain solutions. For example, in their investigation of the friction conditions along the tool face, Oxley and Hastings [54] were forced to make assumptions about the thickness of the deformation zone and the contact length between the tool and the chip. Moreover, the know solutions ignore one of the most basic experimental facts that the chip emerges from the deforming zone with significant angular velocity.

Deriving and analyzing the mixed stress and velocity boundary conditions in the deforming zone, Dewhurst was able to construct his slip line field justifying these conditions (Fig. 23a) and corresponding velocity hodograph (Fig. 23b). Analyzing this velocity hodograph, Dewhurst concluded that the slip-line field can satisfy a range of values of rake angle. Therefore only three conditions $F_1 = F_2 = M = 0$ (the force components that maintain chip equilibrium) remain to determine the value of four parameters ψ, θ, η, and p_A (which is the hydrostatic pressure). As a result, infinity of solutions is likely to exist for every value of the rake angle and shear stress τ. Solving the equations for the force components and moment for any given value of p_A, Dewhurst obtained the relationships between slip-line field angles and hydrostatic pressure (example is shown in Fig. 24). Further, Dewhurst concluded that **the major theorems of engineering plasticity cannot be applied to analyze machining** due to the lack of defined boundary conditions. Also, if strain hardening is included, the solution is in principle poorer since the Uniqueness theorem then does not apply to any steady-state process.

In our opinion, the positive features of Dewhurst's paper are:

- For the first time, a complete set of the boundary conditions was analyzed and the model (Fig. 1.23a) was built using the results of this analysis.
- For the first time, the discontinuity ('jump') of the tangential velocity was presented in the velocity diagram (hodograph). As seen from Fig. 1.23b, the points along slip line *ABC* belonging to the workpiece have the same velocity, *v* (*a*,*b*,*c*), and the same points belonging to the chip have different velocities, namely *a'*, *b'*, and *c'*. For example, *aa'* on this diagram represents the discontinuity of the tangential velocity at point *A* of the model (Fig. 1.23a). Unfortunately it was not noticed by the subsequent researchers.
- The chip was considered to be "born" curved so that the angular chip velocity was included in the considerations.
- The degrees of freedom of the chip formation model are considered not intuitively as it was done by Hill, but using a combination of the model and the velocity diagram.

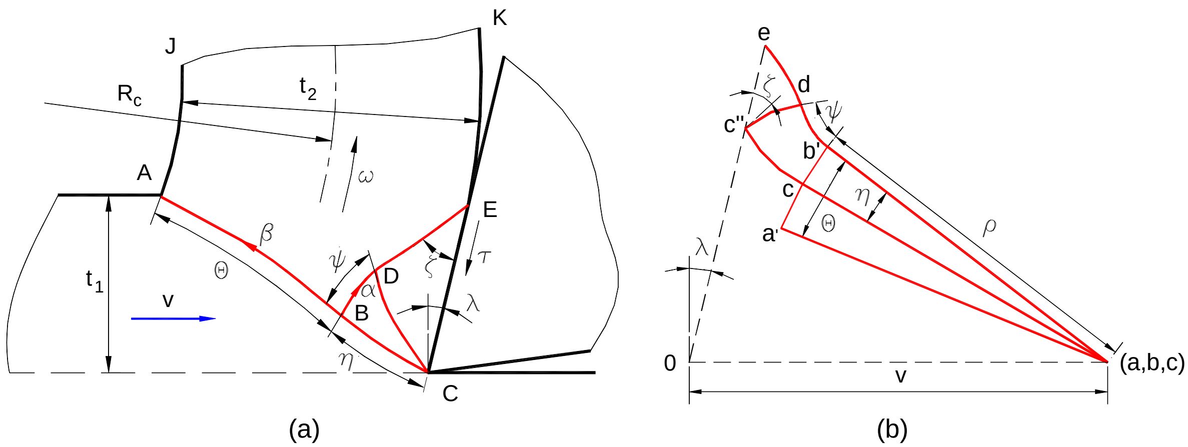

Fig. 23. (a) Slip-line field constructed by Dewhurst defining a class of solutions for any fixed values of the frictional shear stress τ and rake angle λ; (b) Velocity diagram (hodograph).

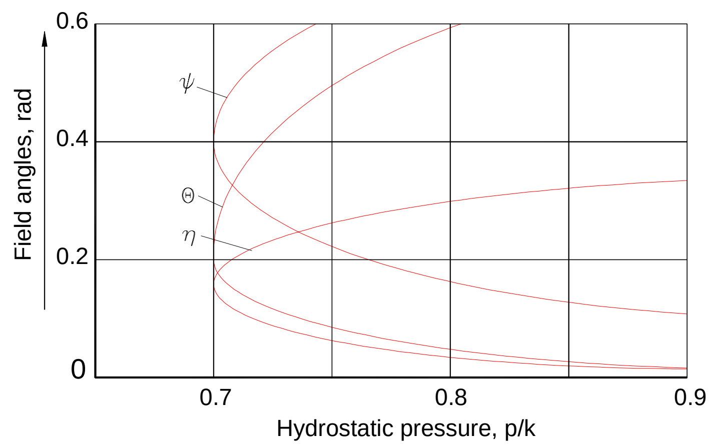

Fig. 24. Relation between clip-line field and hydrostatic pressure for τ = *0.5k* and ζ = ¼π.

The drawbacks of Dewhurst's paper are:

- An idealized rigid-perfectly plastic work material was considered. As a result, the stress at any point should not exceed the yield strength of the work material. The known experimental relationships between the normal and shear stress, peculiar to real work materials, were not utilized.
- All the deformation of the work material takes place along the slip line *ABC* which separates the undeformed work material from the chip. As a result, when an infinitesimal element of work material crosses the shear plane *ABC*, it rapidly sheared into the chip. In this process it acquires that the velocity, which makes it move up the tool and the stress falls to zero. This rapid onset of deformation and velocity occurs at the edge *ABC* of the slip-line field and thus the constructed slip-line field is fictitious because no plastic deformation is allowed above the shear plane *ABC*. In our opinion that was already mentioned in the analysis of Lee and Shafer model, a slip-line field makes sense if and only if it is used to describe strain development in a deformation zone. The model considered by Dewhurst (Fig. 23a) where no plastic deformation allowed in the slip-line field, has no physical difference from that by Ernst and Merchant [33].
- The model assumes that the transition surface does not join up smoothly with the chip free surface (point *A* in Fig. 23a). It was already mentioned, that Zorev criticized Briks model for the same 'sin' explaining what kind of problems it would cause.
- Although the discontinuity of the tangential velocities along slip line *ABC* is recognized, the stress and strain singularities due to this discontinuity are not considered.

- Although the chip is considered as curved from slip line *ABC*, there is no reason (stress or strain distribution) provided to support this assumption.
- The friction conditions along the tool/chip interface is considered (referring to Oxley and Hastings [54]) to be constant and equal $\tau < k$. By the time Dewhurst (as well as Oxley and Hastings) published his paper, it was conclusively proven (and was already included in students' textbook, for example Fig. 2.18 in [17]) that the tool/chip interface consists of two approximately equal parts known as the regions of plastic and elastic contact (or sticking and sliding), respectively. As such, within the region of plastic contact, the shear stress is constant but can exceed *k* significantly (for example, when seizure is the case). Within the second region, the shear stress decreases becoming zero at the end of the tool/chip contact.

To conclude the consideration of Dewhurst paper, we have to point out that it was a significant step ahead in trying to apply engineering plasticity to solve the metal cutting problem. Unfortunately, Oxley [24] criticizing this paper (and thus arguing for the uniqueness of machining), did not point out (and thus did not account for in his book) that the difference between the degrees of freedom of the cutting system and the number of available conditions that is the main cause for non-uniqueness according to Hill and Dewhurst. Presenting the velocity diagram (Fig. 23b) by Dewhurst in his book, Oxley did not notice the discussed above velocity discontinuities on this diagram. As a result, the velocity diagram qualitatively constructed by Oixley and having no such discontinues makes much less sense.

Subsequent researchers argued for the uniqueness of the machined process. Instead of proving that Hill and Dewhurst considerations are wrong or assumptions are too rough, they conducted an experimental study using a 'wide' range of machining parameters and concluded that the discussed non-uniqueness was not observed (**how it supposes to be "observed"? *auth*.**) in their experimental results. They did not bother that, in their experiments [55], the average (the base of averaging customarily is not shown in such publications and one may wonder what it is: a millisecond, a second, a year, or the last one handed years of metal cutting history) values of machining parameters have been measured while the instant values have been discussed by Hill and Dewhurst.

Madhavan, and Chadrasekar tried consider machining as a wedge indentation forgetting that the chip does not form in such an indentation [56]. The computational, experimental, and even theoretical studies of uniqueness are presented. The following objection to this study can be listed as:

- Machining was considered as one of the deforming processes. The authors should be aware that any deforming process preserves the volume of the workpiece. In machining, the machined workpiece and the chip are results of the process. Because these two are separated bodies, the process of such a separation is known as **FRACTURE** [57]. This fracture cannot be accomplished by shearing as stated by Madhavan, and Chadrasekar.
- There is no particular model of chip formation considered so the necessary assumptions associated with such a model are not discussed. It was indicated, however, that friction and adhesion at the chip/tool interface, work material hardening, and temperature and strain rate effects are not considered. The work material was assumed to be rigid-perfectly plastic having the yield shear strength of $350\ N/mm^2$.
- In the theoretical study, the velocity discontinuity [v] in Eq. (1)[56] is misinterpreted as the cutting velocity in Eq. (5)[56] that led to incorrect conclusions.

To conclude the discussion on uniqueness of machining, we would like to point out the following. Mathematics as well as its most advanced numerical methods (like finite element modeling) are no more than a powerful tool that requires very skillful hands to handle. Successful design still requires expert knowledge and intuitive "feel" based on experience; it requires engineers steeped in the understanding of existing engineering systems as well as in the new systems being designed.

With a computer model, however, analysis can be made quickly. The computer's apparent precision - six or more significant figures - can give engineers "an unwarranted confidence in the validity of the resulting numbers." However the question about who makes the computer model of the calculated matter is of more than passing interest. If the model is worked out on a commercially available analytical program, the researcher will have no easy way of discovering all the assumptions made by the programmer. Consequently, the researcher or designer must either accept on faith the program's results or check the results - experimentally, graphically, or numerically - in sufficient depth to be satisfied that the programmer did not make dangerous assumptions or omit critical factors and that the program reflects fully the subtleties of the designer's own unique problem.

Modeling of cutting process using sophisticated finite-element programs become quite popular and many researchers (and unfortunately, papers' reviewers) try to refer to the results of such a modeling as to something conclusively proven. It should be very clear that software incorporates many assumptions that cannot be easily detected by its users but that affect the validity of the results. There are a thousand points of doubt in every complex computer program. Successful computer-aided modeling requires vigilance and the same visual knowledge and intuitive sense of fitness that successful researchers have always depended on when making critical conclusions. If we are to avoid calamitous conclusions, it is necessary for researchers to understand that such errors are not errors of mathematics or calculation but errors of engineering judgment based on the lack of understanding the physical backgrounds, particularly machining.

Unfortunately, some researchers do not seem to be aware of the simple fact that mathematics cannot create physics, physical bodies or phenomena, just as it cannot create biology, life, love, money, or the economy of a country, though it may be very helpful in all these endeavors. Mathematical models, including FEM models cannot output more physics than that was put into them. If a mathematical model used in computer-based simulation of machining is not based on the correct physical model then the results of such a simulation have no value.

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