

# METAL CUTTING THEORY - MISSED CHANCES OR A SCIENCE WITHOUT HISTORY: Part 1

Personal vision of Viktor P. Astakhov

## 1 INTRODUCTION

*"History -a narrative of events; a story. A chronological record of events, as of the life or development of a people or an institution, often including an explanation or commentary on those events"*

The American Heritage® Dictionary of English Language, Third Addition

Each science has its own history that marks the landmarks and milestones of its development. Knowing such a history, a specialist in the field should be able to determine the mainstream direction in its further development avoiding the known dead-ends. For example, the history of machine tools includes a book on each particular machine (e.g., [1]), history of strength of metals [2] includes historical and technical developments over the years as well as biographies of contributors. Biographies and history of developments in the gear design area presented in the Gallery of Fame at the Gear Research Laboratory of the University of Illinois in Chicago and described in many books, e.g. in [3]. Unfortunately, this is not the case in metal cutting. Old papers on metal cutting (when the content rather than the journal space was not of prime concern) took into consideration a number of known works. However, only works relevant or suitable by the authors' opinion to the topic are considered (an exception is the excellent paper by Finnie [4] which will be discussed later). New papers, due to the space limitation in journals, have no room to discuss the place of each particular paper in the development of metal cutting. Moreover, if one hopes to learn the history of metal cutting from the metal-cutting books he/she will be rather disappointed. The following examples support this point.

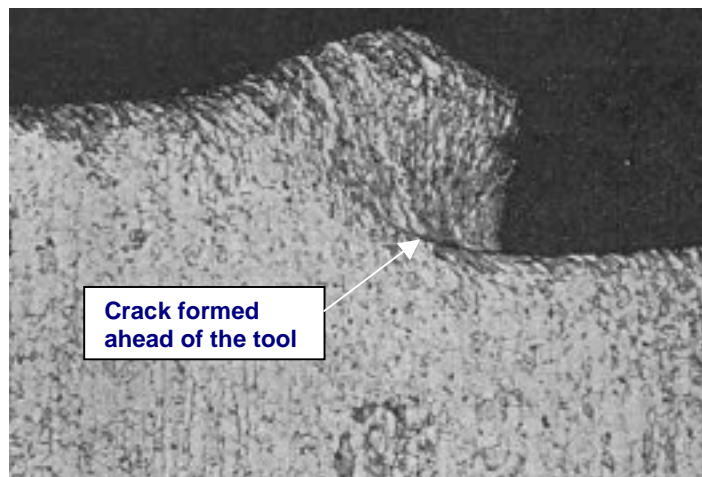
Paul H. Black's book "Theory of Metal Cutting" [5] begins with Section "Metal Cutting – Art to Science." In this section, the history of metal cutting is thought of as started 600,000 – 1,000,000 years ago somewhere in Tanganyika, Africa. After mentioning drilling technique development in Ancient Egypt (about 4000 B.C.), Black briefly listed advances in metal cutting made in the 19<sup>th</sup> century pointing out that the first attempts to explain chip formation were made by Time [6] in 1870 and by Tresca [7] in 1873. Black wrote, "The latter investigator concluded wrongly that the cutting process was one of compression of the metal ahead of the tool. Time in 1877 (ref. [8], *auth.*) reported again that the chip is formed by shear of the metal rather than, as Tresca believed, by it being compressed." In our opinion, this is a very misleading statement. It is true that plastic deformation of any ductile material is accomplished due to its shearing. However, compression is the cause of this shearing and thus no contradiction can be observed between the works of Tresca and Time [9]. We may suggest that there was no theory of engineering plasticity available for Tresca and Time and thus they might not have known what the cause and what the consequences of metal deformation were. However, this information was readily available for Black in 1961.

Furthermore, Black [5] wrote: "A backward step was taken in the publication by Reuleaux in 1900 (ref. [10], *auth.*) when it was reported that a crack was formed in the metal ahead of the tool so that the chip was formed by a splitting operation. This created a misconception long after the 'crack theory' was disproved." It was not explained why Reuleaux's result was a step backwards (**from which reference and established by whom?**) or **who, when and how** disproved this result. In our opinion, Black, making the statement about "a backward step taken in the publication by Reuleaux", just simplified the *section* "A Misconception" of Finnie's paper [4]. It is stated in this paper (Section "A Misconception") that in 1900 Reuleaux [10], the famous German engineer, reported that he had seen a crack ahead of the tool and concluded that the cutting process was similar to splitting wood. This was conformed by observations made by Kingsbury [11], who claimed that a crack ran ahead of the tool. Cutting fluids (coolants) were apparently reaching the point of the tool and it was felt that this would be impossible without a crack. Finnie [4] stated further that "crack" idea was immediately refuted by Kick [12] in a paper a year after Reuleaux's. Kick pointed out what Reuleaux had seen was probably an optical illusion and drew attention to the work of Time [8]. Experiments were made by Kick to show that there was no crack ahead of the tool.

In our opinion, the section "A Misconception" in the Finnie's paper [4] does not appear to be very convincing. First of all, it does not follow from the discussed section under which cutting conditions Reuleaux and Kingsbury observed cracks, what were the work material(s) and tool geometries, what kinds of coolants were used,

etc. Without knowing these data, it is impossible to decide how conclusive are Kick's results. One may wonder if Kick used the same conditions, the same technique of sample preparation and enchants, etc. Our analysis of Kick's paper shows that he did not even use proper experimental techniques and his conclusions were rather qualitative (it is not possible because it could never be possible).

It is interesting to note that in Fig. 1 of the very same Finnie's paper [4] a crack ahead of the tool can be clearly seen and this is not "optical illusion." (Fig. 1) The fact that in Kingsbury experiments the coolant reached the point of the tool was not illusion either. Moreover, the fact that a crack may occur in the front of the tool does not make the cutting of metals analogous to that of wood as Finnie stated [4]. We have to point out here, however, the time when Finnie's paper [4] was written was very special in the history of metal cutting. It was the time when the theory of engineering plasticity developed rapidly and the general impression was that the metal cutting problem would be solved soon using this theory. Because "the crack" was a disturbing factor that makes it impossible to apply the theory of engineering plasticity in metal cutting, the researchers of this time "closed" their eyes and minds to obvious facts that can be observed experimentally.



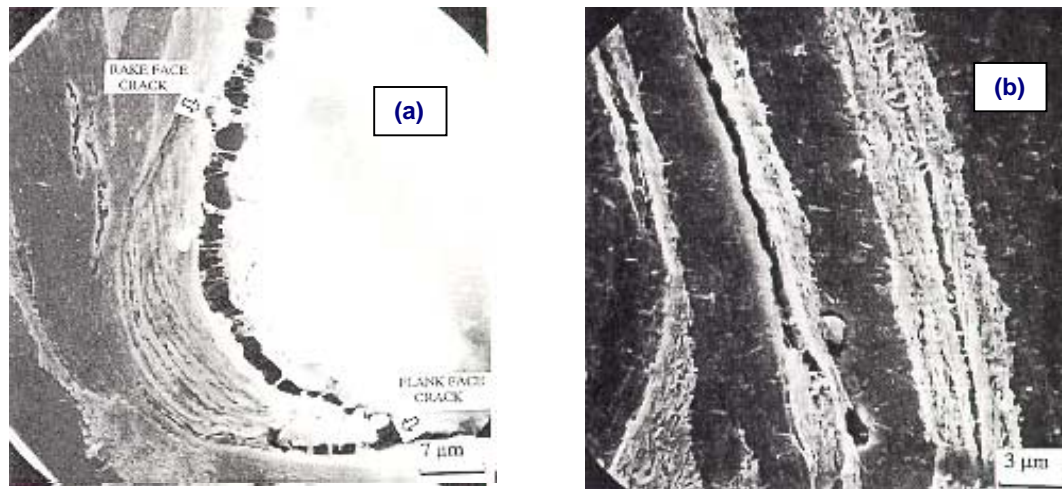
**Figure 1. Photographs of partially formed discontinuous chip (From Finnie, I., Review of the Metal-Cutting Analyses of the Past Hundred Years, *Mechanical Engineering*, 1956, 78, pp. 715-721. With permission).**

Later on Shaw [13,14], studying an elastic-plastic finite element stress field based on an assumed continuum and experimentally observed chip geometry and cutting forces, has found it to be inconsistent with physical conditions that must pertain along the shear plane (constant stress on the shear plane equal to the shear flow stress of the heavy pre-strain hardened work material). Shaw has concluded that the material does not behave as a continuum and that microcracks along the shear plane play a significant role just as they do on the tool face. Although this very important finding explains many known contradictory results, it has not been noticed by the further researchers.

When more sophisticated technique emerged, the presents of cracks in chip formation was conclusively proven in machining of wide variety of work material at macro and micro levels [15,16]. Conducting a very detailed study of chip formation, Itawa and Ueda [15] proved that the continuous chip forms only under relatively specific ( or, exotic) cutting conditions such as when pure single crystal aluminum is machined [15]. Under common cutting conditions, rack(s) is the real phenomenon in chip formation which is classified to be:

- Quasi-continuous chip formation that takes place in machining ductile materials such as steels under favorable cutting conditions. As such crack occurs along the shear direction.
- Discontinuous chip formation that occurs typically when machining brittle materials. As such, the crack nucleated below the flank face and propagate ahead of the cutting tool due to void coalescence;
- Chip formation with built-up edge that takes place in machining "materials which can adhere to the tool face." As such, crack forms initially below the flank face and then ahead of the tool.

Similar phenomena were observed by Sidjanin and Kovac (Fig. 2a) [16]. Besides, since most of the work materials are alloys and thus have different phases and inclusions, cracking in metal cutting occurs between different phases and voids as shown in Fig. 2b [16].



**Fig. 2. Cracks observed ahead of the cutting tool (a) and between different phases of the work material [16].**

Boothroyd [17] started his version of metal cutting history describing achievements of R. Reynolds (1760) and J.Watt (1776), who had a little to do with the theory of metal cutting. Then the author points out that Time and Tresca were the first who tried to explain how chips are made. However, Boothroyd did not elaborate on what they really did. Boothroyd states that as early as 1881 Mallock suggested correctly (in the author's opinion, a typical example of the "personal" bias) that the cutting process was basically one of shearing the work material to form the chip. Then, Boothroyd pointed out "a step backwards in the understanding of the metal cutting process was taken in 1900 when Reuleaux suggested that a crack occurred ahead of the tool". Although it may appear as the shortened version of Black's [5] statement about Reuleaux's results, it does not, simply because it refers to "understanding", i.e. an impression may be created that now we have full understanding of this process and thus we can judge that the Reuleaux's observation and suggestion are wrong.

A significant space in the history section of Boothroyd's book [17] is dedicated to the achievements of Taylor who published his well-known paper in 1907. Although this paper had enormous practical significance, it has little contribution to the metal cutting theory. Then, according to Boothroyd, metal cutting history had stalled until 1941 when Ernst and Merchant published their fundamental work. Because no further development is indicated in the book, one may get the impression that the end of the metal cutting history is reached at this point.

Zorev [18] presenting his vision of the metal cutting history, mentioned but did not consider the Time model, the Merchant force model, and the Lee and Shafer model, etc., i.e., those models that did not "suit" his idea of metal cutting. However, Zorev pointed out that the known single-shear plane model was developed by Zvorykin in 1896 [19] and was justly criticized by Briks in the very same year [20].

Armarego and Brown in their book [21] presented a short two-page description of the metal cutting history.

Shaw's book [22] contains as many as four rows in the history section referring readers to earlier books mentioned above.

Trent in his book [23] on metal cutting, section "Historical", presented a very brief history of manufacturing and did not touch the history of metal cutting at all.

Oxley in his book [24], section "Historical" simply refers to Shaw [22] and Zorev [18].

Boothroyd and Knight in their textbook [25] discussed the history of metal cutting only in the brief introduction to the chapter on the mechanics of metal cutting and this discussion is actually a shortened version of that presented by Boothroyd earlier [17].

Gorczyca [26] in his book entitled "The Application of Metal Cutting Theory," in the introduction to Chapter 4 (only one chapter that deals with the theory) considers Newton's Laws rather than history of metal cutting.

Stephenson and Agapiou in their book on the theory and practice of metal cutting [27] spent a number of pages discussing historical development (Section 1.2). The authors, however, discuss the history of manufacturing, machine tools, etc. paying little attention to the history of metal cutting.

Astakhov [9] in his short section “Historical” briefly analyzes the so-called ‘modern’ metal cutting history, which began in his opinion in 1944 when Merchant published his model of chip formation. Although further discussion in the book contains the historical development of the chip formation model, one can hardly have a complete overview of the history of metal cutting because it is scattered over all of the book’s chapters.

Trent and Wright in their book [28] which is meant to be a textbook spent 1.5 pages on “Short History of Machining” where no one historical fact in the machining process development as well as no one metal cutting specialist is mentioned.

Althintas in his book [29], which is also intended to be a textbook, did not consider the history of metal cutting at all.

Childs, Maekawa, Obikawa, and Yamane in their book [30] did not even mention that metal cutting has any history.

Reading these books, one may wonder if metal cutting has a history at all (besides one exception which is Finnie’s paper [4]). To illustrate that metal cutting does have a history we may refer the reader to the above-mentioned Finnie’s paper [4] and to ASME bibliography book [31] prepared by the *Special Research Committee on Metal Cutting data and Bibliography*. The latter covers the years 1864-1943 and contains 4124 references. Unfortunately no one of the above-discussed books cites this valuable source as a reference. Almost the same can be said about the Finnie’s paper [4], which is seldom referred.

This paper is not intended to present the complete history of metal cutting because the author believes that this history should not be written by a single person in order to avoid personal ‘bias’. Rather, it is intended to analyze the missed chances in the metal cutting history, which are, in the author’s opinion, important to understand the historical development in metal cutting. Each one of these chances could have turned the history of metal cutting significantly so that it could have advanced in different manners if specialists were more concerned with the achieved results rather than protecting ‘convenience’ of the known theories.

## 2 THE BRILIANCE AND POVERTY OF THE SINGLE-SHEAR PLANE MODEL

When one tries to learn the basics of the metal cutting theory, he/she takes a textbook on metal cutting (manufacturing, tool design, etc.) and then reads that the single-shear plane model of chip formation constitutes the very core of this theory. Although a number of other models are known for specialists in this field, the single-shear plane model survived all of them and, moreover, is still the first choice for students textbooks (e.g. [27-30, 32]). A simple explanation to this fact is that the model is easy to teach, to learn, and simple numerical examples to calculate cutting parameters can be worked out for student's assignments. Although it is usually mentioned that the model represents an idealized cutting process, no information about how far this idealization deviates from reality is provided.

More sophisticated readers can learn that there are a number of different models of chip formation and he/she may wonder which one of the known models of chip formation better suits the experimental results. Although the answer is not that simple, we may say that the single-shear plane model better than other known models matches the experimental results (the reason for that is well explained in [9]). It is also interesting that this model was historically the first model developed, then was rejected, and then finally widely accepted remaining ‘a paramount’ today.

The single-shear plane model and practically all its ‘basic mechanics’ have been known since the last century and, therefore, cannot be referred to as the Merchant model. This fact was very well expressed by Finnie [4] who pointed out that while the work of Zvorykin and others, leading to the equations to predict the shear angle in cutting, had relatively little influence on subsequent development, the very similar work of Merchant, Ernst and others almost 50 years later has been the basis of most of the present metal-cutting analyses. In our opinion, if the researchers of that time had the history of metal cutting, there would be no need to re-discover’ the known solutions for the single shear plane model.

The model has been constructed using simple observations of the metal cutting process. Time [6,8] assuming that in metal cutting the state of plane strain exists when the width of cut is considerably greater than the thickness of the layer to be removed, proposed a model of chip formation having a single shear plane as shown in Fig. 3. Zvorykin [19] provided physical explanation for this model. According to this model, the layer to be removed of thickness  $t_1$  transforms into the chip of thickness  $t_2$  as a result of shear deformation that takes place along a certain unique plane  $AO$  inclined to the cutting plane at an angle  $\varphi$ . The velocity relationship between the cutting velocity,  $V$  and the chip velocity,  $V_f$  has been also established in the form as used today.

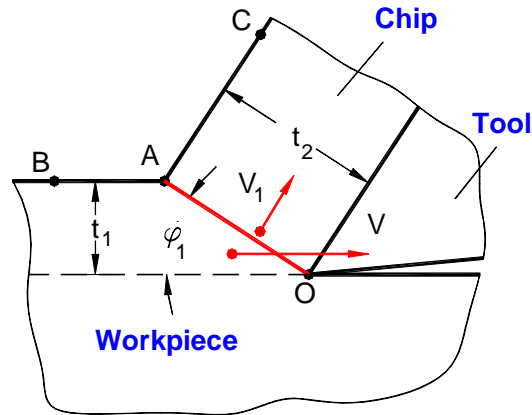


Fig. 4. Single shear plane model.

Although the discussed work became known in Europe and further European studies on metal cutting referred to these works, they were somehow completely unknown in North America where theoretical studies on the metal cutting theory began years later [31]. We may suggest that this was the first missed chance in the history of metal cutting.

As early as 1896 Briks [20] justly criticized the single shear plane model pointing out that the drawbacks of this model are: the single shear plane and the absence of the smooth connection in point  $A$  so that the motion of a particle located in point  $B$  into the corresponding location  $C$  on the chip is impossible from the point of physics of metal deformation. According to Briks, the existence of a single shear plane is impossible for two reasons. First, an infinitely great stress gradient must exist in this plane due to instant chip deformation (chip thickness  $t_2$  is usually 2-4 times greater than that of the layer to be removed,  $t_1$ ). Second, a particle of the layer to be removed should be subjected to infinite acceleration passing the shear plane because its velocity changes instantly from  $V$  into  $V_1$ . Analyzing these drawbacks, Briks assumed that they can be resolved if and only if a certain transition zone where changes of material deformation and its velocity take place continuously and thus smoothly exists between the layer to be removed and the chip. Briks named this zone as the deformation or plastic zone (these two terms were used interchangeably in his work). Unfortunately, these conclusions were much ahead of this time so they were not even noticed by the future researchers until the 1950th. Developing the concept of the deformation zone, Briks suggested that it consists of a family of shear planes (Fig. 3). Such a shape can be readily explained if one recalls what type of tool materials was available at the time of the study. Neither high speed steels nor sintered carbides were not yet introduced and thus Brick conducted his experiment using carbon tool steels. As a result, the cutting speed was low so that the shape of the deformation zone shown in Fig. 3 was not that unusual.

The model proposed by Briks solved the most severe contradictions associated with the single shear plane model. Briks suggested that the plastic deformation takes place in a certain zone which is defined as consisting of a family of shear planes ( $OA_1, OA_2, \dots, OA_n$ ) arranged fanwise as shown in Fig. 4. As such, the outer surface of the workpiece and the chip free surface are connected by a certain transition line  $A_0A_n$  consisting of a series of curves  $A_0A_2, A_2A_3, \dots, A_{n-1}A_n$ . as a result, the deformation of the layer to be removed takes place step-by-step in the deformation zone and each shear plane add some portion to this deformation. We should point out that this model was much ahead of the general level achieved at that time.

Zorev [18] analyzing the Briks model did not mention its advantages. Instead, he pointed out the obvious (as became 70 years later) drawbacks of this model:

- *The model assumes that the transition surface does not join up smoothly with the chip free surface, but the tangent to the curve  $A_{n-1}A_n$  at point  $A_n$  forms a certain angle  $\eta_1$  with the tangent to the chip free surface at the same point as shown in figure 2. Therefore, a micro volume of the workpiece material passing the boundary  $OA_n$  must receive infinitely great acceleration. In our opinion, this is not so. If one analyses boundary conditions on the lines  $OA_1$  and  $OA_n$  [9], this contradiction can be easily resolved.*
- *Briks assumed that the shear surfaces are planes and thus lines  $OA_1 - OA_n$  must be straight. As such, the boundary condition on the transition surface  $A_1A_n$  is so that these lines must form equal angles of  $\pi/4$  with the tangents to this surface in the corresponding points  $A_2 - A_n$ . However, this is only possible if the*

shear surfaces are curved. For example, the straight line  $OA_1$  forms an angle  $\delta < \pi/4$  with tangent  $x_1$ ; in fact this angle has to be equal to  $\pi/4$  so to meet the boundary condition. However, it is only possible if the corresponding shear surface is curved so that line  $OA_1$  has the shape as shown in Fig. 3 to meet the free surface at angle  $\delta_1 = \pi/4$ . **In our opinion this analysis is incorrect.** Being a slip line, the shear plane meets the free surface at  $\pi/4$  only for an idealized single crystal. Because any real metallic workpiece material is a polycrystalline, this condition is not necessarily true. Each crystal in a polycrystalline has its own orientation and which slip plane is available is a matter of the crystal orientation with respect to the direction of the applied load. Moreover, because the deformation in the machining zone is not pure shearing [9], this condition is not valid even for a single crystal. Criticizing Briks model, Zorev did not show any metallographic support to his the  $\pi/4$  statement even though his book contains a great number of micrographs.

- The assumption that the plastic deformation takes place due to shearing along one family of shear planes arranged fanwise is in contradiction with the continuity conditions. **In our opinion, Zorev made this statement without any explanation following a common believe on the deformation mode in metal cutting.** In his book, he did not analyze continuity conditions and thus it is not clear what particularly was meant. We have to point out here that an analysis of the common continuity condition written the following common form

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \tag{1}$$

is not straightforward. For example, Oxley [24] and Astakhov [9] analyzing this continuity conditions came to the directly opposite conclusions.

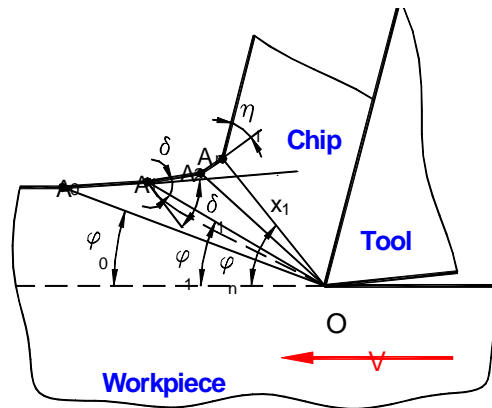


Fig. 4. Briks' model.

Analyzing the Briks' model, Zorev [18] attempted to construct the slip line field in the deformation zone using the basic properties of slip lines. According to his consideration, the deformation process in metal cutting involves shear and, therefore, is characterized by the lines of maximum shear stress, i.e. by characteristic curves or slip lines (**making this statement, Zorev assumed that pure shear deformation is the prime deformation mode and no strain-hardening of the work materials is taking place, auth.**). He considered the deformation zone as superposition of two independent processes, namely, deformation and friction. Considering deformation, Zorev pointed out that it occurs in two different zones A and B as shown in Fig. 5. Utilizing basic properties of shear lines (this term is used in Zorev's book), he constructed these lines in Zone A (Fig. 5) as two families of mutually orthogonal curves. The angle between the tangents to a pair of neighboring curves of one family remains constant as they move these lines. At any point of intersection with the free surface, the tangent to the shear lines and the normal to the free surface form an angle of  $\pi/4$ . This, in Zorev's opinion, is true because a normal to the transition curve (a part of the free surface between the unreformed layer and the chip that form one of the boundaries of the deformation zone) coincides in the direction with one of the principal stresses (**although no explanation to this statement can be found in the book, auth.**).

Zone B (Fig. 5) is thought of as consisting of three regions. The region  $M_1OM_2$  is formed by two families of mutually orthogonal (a new mathematical terms meaning double jeopardy as a result of too direct translation from Russian, *auth.*) straight lines intersecting the face at angle of  $\pi/4$ . Second region  $M_2OM_3$  includes a singular point  $O$  (again, no explanations why, *auth.*) “through which pass a family of straight lines in a form of fan” (if  $O$  is a singular point, no slip line should pass through it, *auth.*). The second family “takes the form of concentric arcs of circles” (no explanations to this suggestion, *auth.*). Assuming the contact along the flank, Zorev suggested that the third region  $M_3OM_4$  is similar to the first one.

Zorev further considered that when friction at tool/work material interfaces is allowed in the model, the friction force change “the characteristic curves” (term used interchangeably with shear lines in the book, *auth.*) and thus a new arrangement of slip lines shown in Fig. 6 is the case in Zone B. Only brief qualitative justification to this picture is provided.

In Zorev’s opinion, his qualitative explanations were sufficient to “imagine” an arrangement of the shear lines throughout the whole plastic zone ‘in approximately the form’ shown in Fig. 7 because their beginnings and ends “coincide with the corresponding slip lines in zones A and B.” In our opinion, one should have a really special imagination to be able to construct shear lines knowing only the inclinations of their start and end points. Moreover, these inclinations could be true if and only if simple shearing is the prime deformation mode in metal cutting. Unfortunately, no experimental evidence to prove this point is presented in Zorev’s book (this condition is not even motioned, *auth.*)

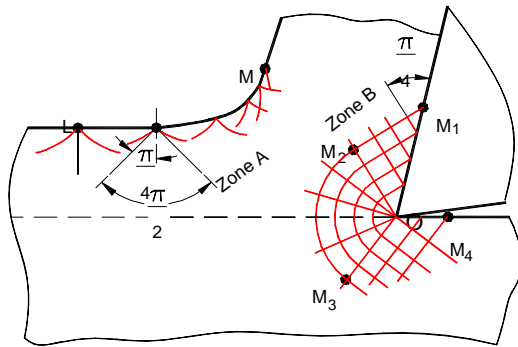


Fig. 5. Slip lines in Zones A and B.

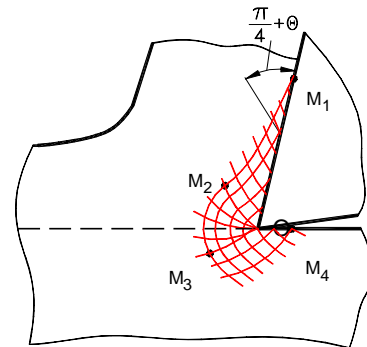


Fig. 6. Slip lines in Zone B when friction on the rake face is considered.

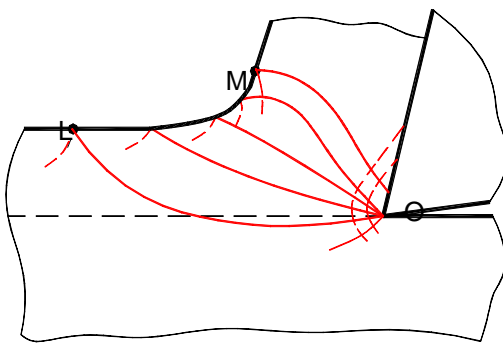


Fig. 7. Zorev’s qualitative model.

According to Zorev, plastic zone  $LOM$  is limited by shear line  $OL$ , along which the first plastic deformation in shear occurs; shear line  $OM$  along which the last shear deformation occurs (what is the principal difference between this line and shear plane  $OA_n$  in the Briks model (Fig. 3) criticized by Zorev?, *auth.*); line  $LM$  which is a deformed section (a line is a deformed section?, *auth.*) of “the other surface of the cut” (in metal cutting literature is known as the free surface, *auth.*). The plastic zone  $LOM$  includes “a family of shear lines along which growing shear deformation are formed successively.” Zorev stated that such a shape of the deformation (plastic) zone is based on the multiple experimental studies and this is more than a very serious statement.

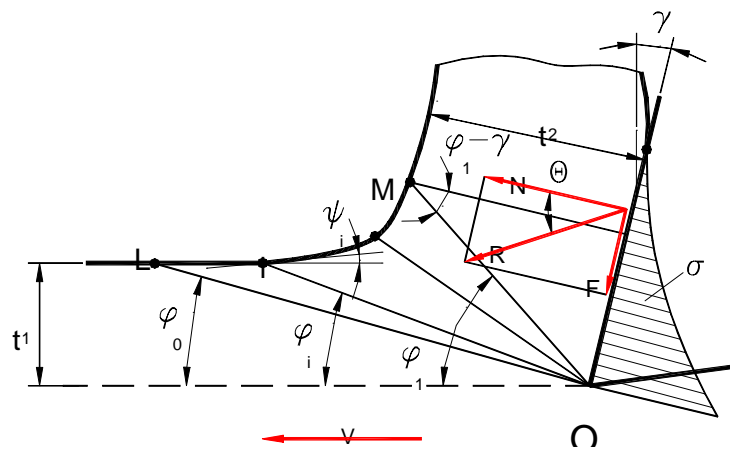
It must be recognized that this work is the most extensive experimental work in the field of metal cutting ever. No other study (known to the author, at least) offers the results of so many well-conducted experiments performed using a number of different work materials, tools, and cutting conditions. It is understood that it is next to impossible to accomplish all these by a single researcher. In reality, the book summarizes the results of the many-year experimental studies performed at the leading in the former Soviet Union research institute, which coordinated manufacturing activities in the whole USSR industry. Professor Zorev was the director of the institute and the chairman of its scientific committee where the main results presented in the book have been discussed in details. As a result, the book should be considered as a scholarly treatment of the Mechanics of Metal Cutting and thus should represent a valuable source for researchers. In our opinion, the book deserves to be considered as the Bible of Metal Cutting. Unfortunately, that is not the case even though each serious study in the field cited it as a reference.

The reason for this is an extremely poor translation and editing of Zorev's book that make it very difficult to follow, particular for a North-American reader who should spend a great deal of time to understand the terms used in the book. For example, the term 'plane chip formation' used instead of the plane strain state; 'tangential stress' instead of the shear stress; 'acute-angle cutting' and 'free cutting at the right angle' instead of orthogonal cutting; 'shear lines' instead of slip lines; 'flexible recoil of the machined material' instead of spring back of the machined material; 'cutting ratio' instead of chip compression ratio which is reciprocal to the chip ratio; 'true angle of cut' and 'true cutting angle' instead of the deviation of the chip sliding velocity from the normal to the cutting edge; 'chip speed vector' instead of chip velocity vector (speed is a scalar); 'relative deformation' instead of strain; 'relative shear strain' instead of true strain; etc. Moreover, the designations of Russian work and tool materials are used throughout the book without pointing out that practically all these materials have *AISI* analogs with which a North American reader is much more comfortable.

Trying to analyze the model shown in Fig. 7, Zorev arrived to a conclusion that there are great difficulties in precisely determining the stressed and deformed state in the deformation zone ('plastic zone' in the book) using the theory of plasticity. This discouraging conclusion was made by a great specialist in metal cutting who worked more than 30 years in the field and had enormous human resources and funding in his possession. He pointed out that the reasons for this conclusion are:

- The boundaries of the deformation zone are not set and thus cannot be defined. In other words, there is not steady-state mode of deformation in metal cutting; the deformation zone has an ever-changing shape.
- The stress components in the deformation zone do not change in proportion to one another. In modern interpretation it means all but a non-linear deformation mode so that simple shearing is not adequate to describe deformation in metal cutting.

As a result, Zorev was forced to consider an approximate stressed and deformed state of the deformation zone. In doing this, he adopted a simplified model shown in Fig. 8. This model differs from that in Fig. 7 in that the curves of the first family of shear lines are replaced by straight lines and, in addition, it is assumed that there is no shear along the second family of shear lines adjacent to the tool rake face.



**Fig. 8. Simplified Zorev's model with a smooth chip free surface.**



Zorev admitted that he finally arrived to the Time model (Fig. 2) [6,8] and using a simple geometrical relationship that exists between two right triangles  $OKP$  and  $ONP$  obtained the Time formula for chip compression ratio  $\xi$

$$\xi = \frac{\cos(\varphi_{sp} - \gamma)}{\sin \varphi_{sp}} \quad (7)$$

Assuming further that  $\varphi_1 \approx \varphi_{sp}$  using Eq. (3), Zorev wrote

$$\varepsilon_1 \approx \varepsilon_{sp} = \cot \varphi_{sp} + \tan(\varphi_{sp} - \gamma) \quad (8)$$

It follows from Eqs. (7) and (8) that

$$\varepsilon_1 \approx \varepsilon_{sp} = \frac{1 - 2\xi \sin \gamma + \xi^2}{\xi \cos \gamma} \quad (9)$$

Assuming that  $x_l \approx t_l$  and using the force diagram shown in Fig. 8, Zorev obtained the simplified expression for determining the shear stress on the final boundary of the deformation zone as

$$\tau_1 \approx \tau_{sp} = \frac{R \cos(\varphi_{sp} + \omega) \sin \varphi_{sp}}{t_l b} \quad (10)$$

where  $R$  is the cutting force;  $b$  is the width of cut. Zorev mentioned that because  $\varphi_{sp} < \varphi_1$  and  $t_l < x_l$ , Eqs. (9) and (10) give somewhat ‘enhanced’ values for deformation and stress (how much “enhanced”? Compare to what?, auth.).

To accomplish his complete arrival to the single shear plane model, Zorev mentioned that at high cutting speeds (how high?, auth.) the deformation zone narrows considerably so that its final boundary approaches the specific shear plane. Therefore,  $\varphi_{av} = \varphi_{sp}$  can be assumed in Eq. (6)

$$\beta_1 = \varphi_{sp} + \text{arc cot} \left( \frac{\varepsilon_1}{2} + \sqrt{1 + \frac{\varepsilon_1^2}{4}} \right) \quad (11)$$

Zorev admitted that relationships (7), (9)-(11) are well known in reference sources as derived directly from an examination of the single shear plane model. However, the way they were derived in the Zorev’s book gives more general solution from which other known models can be obtained. This is explained as follows.

Equation (2) written for the specific shear plane takes the following form

$$\varepsilon_{sp} = \cot \varphi_{sp} - \cot(\varphi_{sp} - \psi_{sp}) \quad (12)$$

Zorev further assumed that  $\kappa \varphi_1 \approx \varphi_{sp}$  and  $\varepsilon_1 \approx \varepsilon_{sp}$  and thus Eq. (4) becomes

$$\varepsilon_{sp} \approx \cot \varphi_{sp} - \tan(\varphi_{sp} + \omega) \quad (13)$$

From Eqs. (12) and (13) it follows that

$$\cot \varphi_{sp} - \cot(\varphi_{sp} - \psi_{sp}) \approx \cot \varphi_{sp} - \tan(\varphi_{sp} + \omega) \quad (14)$$

from which

$$2\varphi_{sp} + \omega \approx \frac{\pi}{2} - \psi_{sp} \quad (15)$$

or finally

$$2\varphi_{sp} + \theta - \gamma \approx \frac{\pi}{2} - \psi_{sp} \quad (16)$$

Zorev showed that all the known solutions for the specific shear angle can be obtained from this equation.

For the single shear plane model the tangent drawn to the workpiece free surface at point  $P$  (Fig. 9) is a horizontal line and thus  $\psi_{sp} = 0$ . Substituting this value into Eq. (16) we obtain

$$2\varphi_{sp} + \theta - \gamma = \frac{\pi}{2} \quad (17)$$

which is the know Ernst and Merchant solution [33]. Using the notations  $\psi_{sp} = c_l$  and  $(\pi/2 - \gamma) = \delta$ , the known Zvorykin solution [19] is obtained

$$\varphi_{sp} = \frac{\pi}{2} - \frac{\theta + c_1 + \delta}{2} \quad (18)$$

Using the notations  $\psi_{sp} = c_1$  and  $(\pi/2) - c_1 = c$ , the modified Merchant solution [34] is obtained

$$2\varphi_{sp} + \theta - \gamma = c \quad (19)$$

Substituting  $\psi_{sp} = \theta - \gamma$  into Eq. (16), the Lee and Shafer solution [35] is obtained

$$\varphi = \frac{\pi}{4} + \gamma - \theta \quad (20)$$

Substituting  $\psi_{sp} = \theta$  into Eq. (16), the Stabler's formula is obtained

$$\varphi = \frac{\pi}{4} + \frac{\gamma}{2} - \theta \quad (21)$$

and so on.

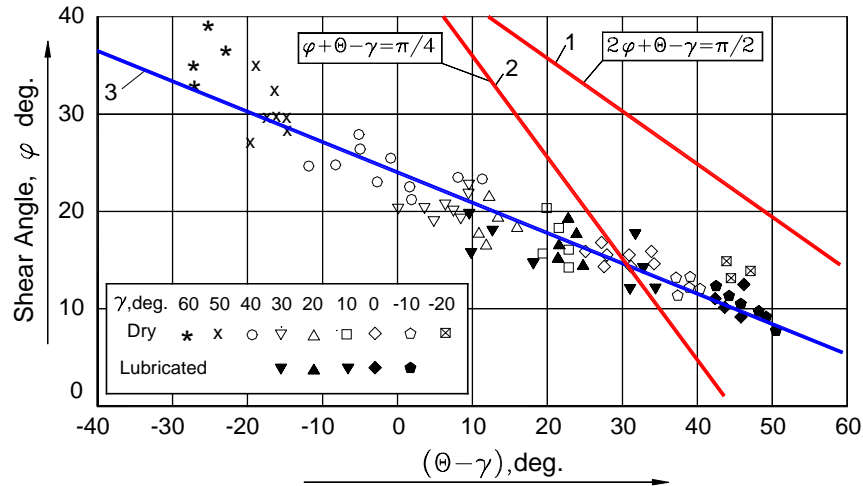
Analyzing these results, Zorev came to the conclusion that all solutions related to Eq. (16) are formal. In other words, **they contain neither physics nor mechanics of metal cutting although no one work admits this fact.** Moreover, all the discussed results obtained using a simple geometrical considerations performed in the 19th century by Time and Briks and has little to do with the physics or even mechanics of metal cutting because no physical laws (besides the law of simple friction of the chip at the tool/chip interface and the applicability of this law has always been questioned [9]) and/or principles of mechanics of materials have been utilized in the course of the discussed solutions. Nonetheless, it is a common belief that the discussed solutions provide 'physical inside into the cutting process' (for example, [27], p.464).

The next logical question about how good is the single shear plane model should be considered here. In other words, how far is this model from reality. Naturally, during the period of 1950–1960, when decent dynamometers and metallographic equipment became available, a number of works were done to answer this question. In our opinion, the results these researches are well summarized by Pugh [36] and Chisholm [37].

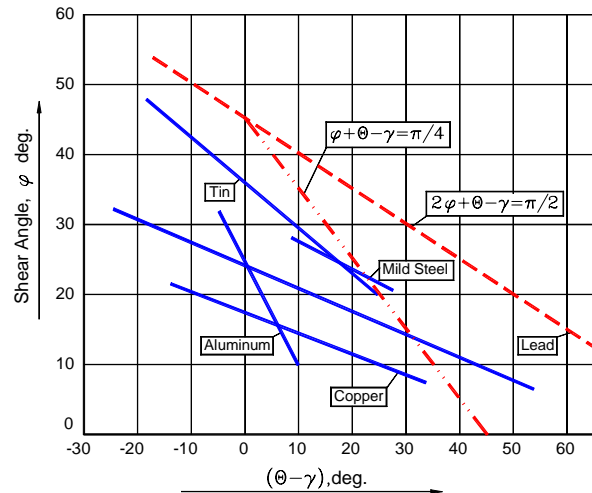
In our opinion, the best research results with a detailed description of the experimental methodology is presented by Pugh [36]. In his study, all the possible 'excuses' for 'inadequate' experimental technique were eliminated. The experiments were carried out in two basic phases:

- The first phase included pure orthogonal cutting at a low cutting speed (to eliminate the possible influence of the cutting temperature and strain rate) on a planing machine. The tests were carried out on lead, tin, and aluminum, and in each case metal specimens in the form of blocks 5 in. (127 mm) long,  $1\frac{3}{4}$  in. (44.45 mm) wide and 2 in. (50 mm) deep were machined with tools of rake angle varying from  $-20^\circ$  to  $60^\circ$ . The clearance angle of  $5^\circ$  was maintained throughout the experiments. Unlubricated cuts ranging in depth from 0.01 in. (0.254 mm) to 0.2 in. (5.08 mm) were taken at speeds varying from 1 in. to 6 in. per minute (0.0254 – 0.1524 m/min).
- The second phase was carried out at high cutting speed on a lathe. Lead, mild steel, and high-conductivity copper were used as work materials. The cutting speed varied from 20 to 580 ft. (6-176 m/min). Both dry and lubricated (with carbon tetrachloride) test were carried out. The analysis of the test results show that the results of the slow speed planing tests do not appear to be markedly different from those of the slow speed lathe cutting tests.

The experimental results are all but conclusively proved that for **every work material tested**, there is marked disagreement in the ' $\varphi$  vs  $(\theta - \gamma)$ ' relation between experiment and the prediction of both the Ernest and Merchant and the Lee and Shafer theories. The example of the experimental results for lead is shown in Fig. 10. Figure 11 shows the results for all the tested materials. As seen, the experimental results are not even close to those predicted theoretically.



**Fig. 10. Relation between  $\phi$  and  $(\theta-\gamma)$  for lead [36]: 1 – Ernst and Merchant solution, 2- Lee and Shafer solution, 3 – experimental results.**



**Fig. 11. Comparison between calculated and experimental results for tin, aluminum, mild steel, lead and copper [36].**

The similar conclusive results were presented by Creveling, Jordon, and Thomsen [37] (an example is shown in Fig. 12) and by Chisholm [38].

The modified Merchant solution (Eq. (19) in which the shear stress is assumed to be linearly dependant on the normal stress through a factor  $k_I$  ( $c = \cot^{-1}k_I$ ) as

$$\tau = \tau_o + k_I \sigma \quad (22)$$

(according to Merchant,  $\tau_o$  and  $k_I$  are work material constants) has been also examined in the case of copper and mild steel. The form of Eq. (19) to fit experimental data requires  $c$  to be  $47^\circ$  for copper and  $69^\circ$  for mild steel. Equation (22) is shown plotted in Figs. 13 and 14 together with the experimentally obtained values for copper and mild steel [36]. It can be seen from these figures that the shear stress does not increase with the normal stress at anything like the rate required to enable the modified Merchant solution to fit experimental results. In fact, it would appear that the shear stress is almost independent of the normal stress on the single shear plane.

The above conclusions were conformed by Bisacre [36] who conducted very similar cutting experiments trying to find a value for  $c$  in Eq. (19). The results of these experiments enabled Bisacre to conclude that if the Merchant solution (theory) were correct, that there would be a marked effect of the normal stress on the shear

stress acting along the shear plane. To support his point, Bisacre noted that the results of tests carried out in which the same material was subjected simultaneously to torsion and axial compression, showed that the shear strength of the material was almost independent of normal stress. As a result, the divergence of the constant  $c$  (Eq. (19) from  $\pi/2$  cannot be attributed to the effect of the normal stress on the shear strength of the work material.

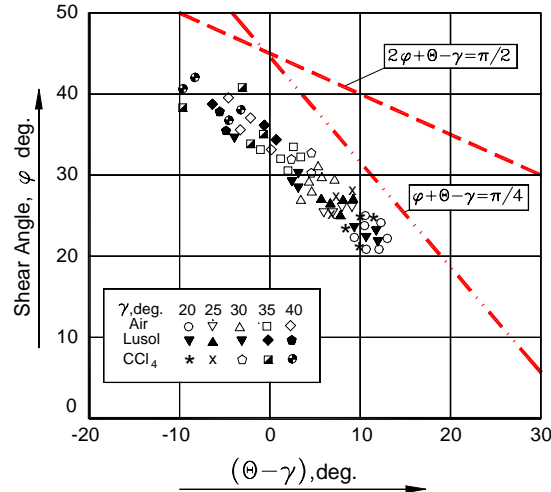


Fig. 12. Relation between  $\phi$  and  $(\theta - \gamma)$  for steel SAE 1113 [37].

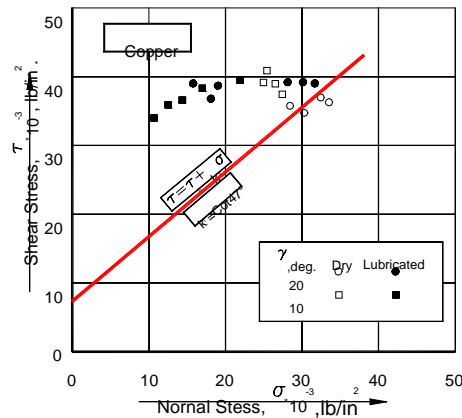
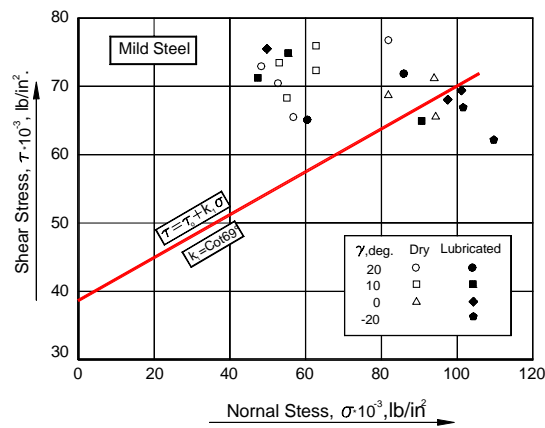


Fig. 13. Comparison between the estimated and experimentally obtained relationship “shear stress-normal stress” for copper [36].

Zorev also presented clear experimental proofs that the discussed solutions are inadequate [39]. In his notation,  $\theta - \gamma = \omega$  which he called the angle of action, that is, the angle between the vector of the chip formation force,  $R$  and that of the cutting speed. As mentioned above, the research institute led by Zorev checked the validity of the Merchant solution (Eq. (19)). It was conclusively proven that this solution is not valid even in the simplest case of cutting with low cutting speeds. It was found that the constant  $c$  in Eq. (19) cannot be considered as a constant. Rather, it varies with the work material properties, cutting regime, tool geometry, etc. Zorev pointed that there is even less foundation for considering Eq. (19) admissible for more intricate cases of cutting when high cutting speeds are used.



**Fig. 14. Comparison between the estimated and experimentally obtained relationship “shear stress-normal stress” for copper [36].**

Reading this, one may wonder why Zorev did not mention his findings about the single shear plane model in his book [18] that had been published 5 years later. In our opinion, if he had done so, he would have recognized that there is no available model of metal cutting at all. As a result, he included the above-discussed ‘general solution’ for the single plane model ‘forgetting’ to mention that none of the possible particular solutions to this model is in any reasonable agreement with experiments.

Thus it is clear that there is a marked disagreement between the above discussed solutions and the experimental results and this disagreement has been conformed by many researches. One might expect that knowing these results a single shear plane model will be just a part of the history. In reality, however, this is not the case and the single shear plane model managed to ‘survive’ all these conclusive facts and still is the first choice for the entire textbook used today. In contrary, all excellent works showing complete disagreement of this model with reality are practically forgotten and not even mentioned in modern metal cutting books [9, 22-30], which still discuss the single plane model as the basic model of chip formation.

Moreover, a book “Application of Metal Cutting Theory” [26] is entirely based on this model showing how to apply it in practical (?) calculations.

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